THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[SEVENTH SERIES.]

JANUARY 1932.

I. *The Charge carried by Atoms of Radium* D *emitted by α-ray Recoil from a Source of Radium* C *on a Metallic Surface, and its Relations with the Surface Forces.*

By J. D. McGEE, *M.Sc., Exhibition of* 1851 *Scholar of Sydney University, Clare College, Cambridge* *.

Introduction.

WHEN an α-particle is emitted by a radioactive nucleus the residual atom recoils with a momentum equal and opposite to that of the α-particle. Its velocity is about 4×10^7 cm. per sec., and it travels a distance of about ·12 mm. in air at atmospheric pressure. Since by the emission of the α-particle the charge of the nucleus is reduced by two units the residual atom will have two superfluous electrons in its outer structure. If these are retained by the new atom it will appear initially with a double negative charge, but they can only be bound to the atom with a negligible energy, and will most probably be dislodged very easily.

An α-particle in escaping from an atom passes right through the extranuclear electrons with which it may experience collisions and so ionize the atom. It is known that an α-particle in passing through a gas produces intense ionization along its path, somewhat less than half of which is due to the expulsion of electrons from gas atoms by the

* Communicated by Lord Rutherford, O.M., D.Sc., F.R.S.

α-particle *. It seems reasonable to suppose that the deeper an α-particle penetrates into the structure of an atom the greater the probability of ionization taking place. On this assumption it can easily be shown that an α-particle must pass within an average distance of 2×10^{-9} cm. of the nucleus of an atom of air in order to ionize it—that is, it must penetrate well into the structure of the atom.

A case analogous to the expulsion of an α-particle by a radioactive atom is that in which the nucleus of a gas atom is involved in a "head-on" collision with an α-particle. In the latter case the α-particle must pass into and out of the atom, whereas in the former it passes out only; but the disturbance produced in each case must be of the same order of magnitude and very similar in nature. It seems highly probable that an atom involved in such a "head-on" collision would lose one if not more electrons and a similar result is to be expected from the emission of an α-particle by a radioactive atom. Some evidence on this point has been obtained by Blackett from photographs of forked α-particle tracks in an expansion chamber †. These represent very close collisions between the α-particle and the gas atom, and the recoil range and initial energy of the latter can be determined. From a consideration of the relative ranges for the same initial energy of such recoil atoms in air, hydrogen, helium, and argon their average charge can be estimated, and is found to be between two and three positive units for the heavier gases.

The initial charge carried by recoil atoms can only be investigated when they are observed under such conditions that they cannot interact with surfaces or other atoms before their charge is determined. Until the recent experiments of Mund, Capron, and Jodogne ‡ the charge on recoil atoms had only been observed after they had escaped from a metal surface on which the parent substance was deposited. In their experiments, however, they appear to have successfully determined the charge carried by radium A recoil atoms in such a way that the effect of interaction with other atoms has been eliminated. They conclude that the radium A recoil atoms carry an initial average charge of two positive units.

In their experiments radon was admitted for a short time (about one minute) into a glass vessel at a pressure of ·001 mm. The glass vessel contained two parallel metal

* *Cf.* Rutherford, Chadwick, and Ellis, 'Radiations from Radioactive Substances,' pp. 143 *et seq.*

† *Loc. cit.* p. 252.

‡ *Bulletin de la Soc. d. Chim. d. Belgique*, no. 1, Jan. 1931.

plates, 1 cm. apart, between which a field of 3200 volts per cm. could be maintained. This field was sufficient to collect an appreciable excess of the recoil atoms on one or other of the plates if they were initially charged, and the pressure was such that not more than 10 per cent. of the recoil atoms made collisions with other atoms before reaching the collecting plates.

They found that the cathode received a larger number of recoil atoms than the anode. Since the mass and initial velocity of the recoil atoms are known, and their distribution in space between the electrodes is uniform, the fraction captured by each electrode can be calculated for any assumed value of their initial charge. They found that the observed distribution of activity between the two plates corresponded to that calculated on the assumption that the initial charge on the recoil atoms was $+2{\cdot}0e$.

This result is of considerable interest, since it shows that the escaping α-particle leaves the radium A recoil atom doubly ionized. In the passage of α-particles through gases doubly ionized atoms are only found in the case of helium *, and then only at the end of the range of the particle. But it is to be noted that collisions of an α-particle in a gas which can be compared with the case of the escape of an α-particle from a nucleus are so rare that, even if they did produce double ionization, their effect would not usually be detected. This conclusion is, however, in good agreement with Blackett's observation that atoms recoiling after a close collision with an α-particle carry two or three unit positive charges.

In radioactive experiments it is customary to use sources in which the active matter is deposited on a metal surface, and it is of considerable importance to know the behaviour of recoil atoms emitted from such a source. The charge carried by the recoil atoms will still be dependent to some extent on its initial state, but it will be subjected to the further influences of the surface forces and any collisions it may make with other atoms before escaping from the surface. In sources prepared in the usual way by exposing a metal surface at a negative potential in radon it is very probable that some of the active matter is embedded in the metal surface. Hence some of the recoil atoms from such a source must make collisions with atoms of the metal. Again, part of the active matter which is deposited on the surface may exist in the form of aggregates or it may be covered by a

* T. R. Wilkins, Phys. Rev. xix. p. 210 (1922).

layer of adsorbed gas. In either case the escaping recoil atoms would interact with other atoms. Finally, the charges carried by recoil atoms of the same type may be different when they escape from the clean surfaces of different metals, since the work-functions vary considerably.

The charge carried by a recoil atom escaping from a source deposited on a metal surface and recoiling *in vacuo* was first investigated by Makower * and his collaborators by bending a beam of α-rays and recoil atoms in a magnetic field. They concluded that the momentum of the recoil atoms was closely equal to that of the α-particles, and that each carried one positive unit of charge. The same result was obtained for radium B atoms recoiling from radium A and radium D atoms from radium C, except that in the latter case the average momentum of the recoil atoms appeared slightly smaller than that of the α-particles. This was attributed to some of the radium C being slightly embedded in the metal surface in the preparation of the source.

A more direct method of measuring the charge on radium D recoil atoms from a source of radium C has been used by L. Wertenstein †. He collected a beam of rays from a radium C source in a Faraday cylinder, and measured the charge received by it per unit time (a) when α-, β-, and recoil rays were received by the cylinder, (b) when the recoil atoms had been stopped by a very thin screen of aluminium, and (c) when both α-rays and recoil atoms were cut out of the beam by a screen of greater stopping power. Now (b–c) gives the charge received by the Faraday cylinder from a certain number of α-particles, and (a–b) gives the charge contributed by the same number of recoil atoms, provided the efficiency of recoil is 100 per cent. Then $\left(\frac{a-b}{b-c}\right) \times 2e$ gives the charge on the recoil atoms. Wertenstein found the recoil atoms to be neutral when the pressure in his apparatus was ·0005 mm. of mercury, determined by a Knudsen absolute pressure gauge, but they became positive when a pressure of ·0025 mm. of gas was admitted. He therefore concluded that the recoil atoms were always neutral initially, and became positively charged only after collisions with other atoms. The distance travelled by the recoil atoms in his experiments was short, about 1·5 cm., so that the chance of a collision with a gas molecule at the

* Phil. Mag. xx. p. 815 (1910); xx. p. 882 (1910); xxix. p. 253 (1915); xxx. p. 811 (1915).

† *C. R. de la Soc. d. Varsovie,* viii. p. 327 (1915).

pressure at which he worked (·0005 mm.) was too small to influence his results.

Neither in Wertenstein's experiments nor in those by Makower and his collaborators were any precautions taken to eliminate the effects on the charge of the recoil atoms of the various factors which have been mentioned above. It is with the separate effects of these factors and the determination of the charge carried by recoil atoms subject only to the electrical forces of the surface from which they come that the experiments to be described are chiefly concerned.

Discussion of the Conditions necessary for the Experiment.

In order to approximate to the ideal conditions of recoil the disintegrating atoms must be deposited *on* the surface and not *in* the metal, as appears to be the case with sources of radium C prepared by recoil in radon, and the surface must be free from adsorbed layers of gases and vapours. Under these conditions it seems reasonable to suppose that a recoil atom which is shot out perpendicular to the surface will be influenced only by the surface forces of the metal.

The experiments by Barton * and Philipp and Donat † have shown that it is possible to obtain up to 20 per cent. efficiency for β-ray recoil when the source is prepared *in vacuo* by distillation and kept *in vacuo* during the experiment. Barton also found that efficiency of β-ray recoil of the same order could be obtained by depositing the source on platinum by electrolysis and afterwards heating it up to 400° or 500° C. for a few seconds in a good vacuum to remove adsorbed vapours and gas from the surface.

Since the β-ray recoil atoms in general have energies of less than 0·5 electron-volts and the α-ray recoil atoms have energies of 10^5 electron-volts, the surface conditions which give a good efficiency of β-ray recoil should be almost perfect for α-ray recoil.

The recoil must take place in a sufficiently high vacuum to ensure that only a very small percentage of the recoil atoms make collisions with gas molecules during their passage from the source to the Faraday cylinder.

The Faraday cylinder itself must be an efficient collector of the rays.

Description of Apparatus.

In order to repeat Wertenstein's experiments with the suggested improvements the apparatus shown in fig. 1 was

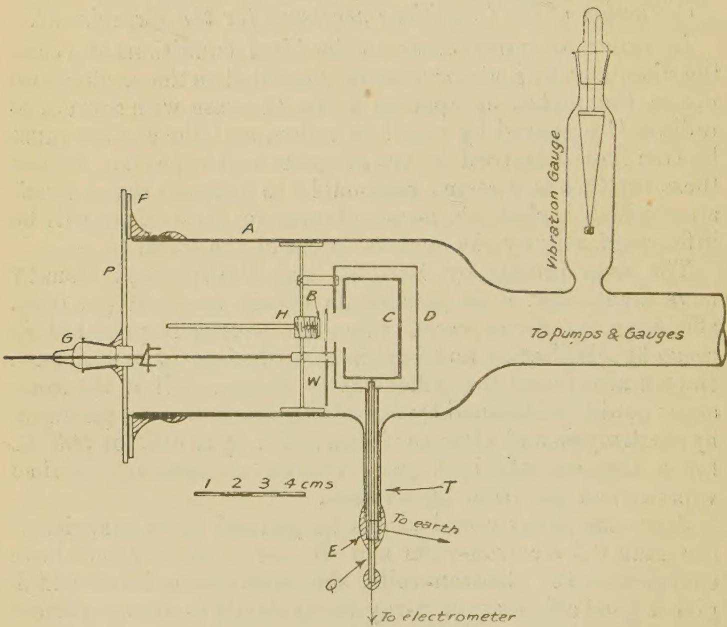
* Phil. Mag. i. p. 835 (1926).

† *Zeit. f. Phys.* xlv. p. 512 (1927); also lix. p. 6 (1930).

constructed. Modifications that were made will be described at the various stages in the work at which they were found necessary.

The apparatus consisted of a glass cylinder A, 6·3 cm. in diameter, closed at one end by a brass plate P, which was ground to fit a brass flange F, which in turn was waxed on to the glass cylinder. The Faraday cylinder C was insulated from the screening cylinder D by quartz rods. The lead from the Faraday cylinder to the electrometer was taken out

Fig. 1.



through the tube T from which it was insulated by a quartz tube Q, which was screened by a surrounding brass tube E. The joints were made air-tight with sealing-wax. The metal disk B, on which the source was deposited, was fixed in front of the opening in the screening cylinder D, and between the two was the wheel W carrying three circular "windows," *a*, *b*, and *c*, slightly greater in diameter than the opening in the cylinder D. This wheel could be rotated by rotating the stopper in the ground-glass joint G. This was cemented through the brass plate P with sealing-wax.

Of the three windows in the wheel W, (*a*) was open, (*b*) was covered by a thin collodion film of stopping power

for α-rays equivalent to 2 to 3 mm. of air, this being sufficient to stop the recoil atoms, but not to absorb the α- or β-rays appreciably. The third window (c) was covered by a sheet of mica of 8 cm. stopping power for α-particles, which was sufficient to stop all the α-rays from radium C, but did not stop more than a small percentage of the β-rays.

The end of the screening cylinder D which faced the source was lined with lead sheet of 1 mm. thickness, so as to limit the beam of rays received by the Faraday cylinder quite definitely. The Faraday cylinder itself was made of brass and lined with lead sheet 1 mm. thick to prevent fast β-rays which enter it from penetrating its walls and escaping.

The lead lining was used for a further reason. Recent experiments by Philipp and Donat * and L. Wertenstein † have shown that the temperature of the collecting cylinder and the metal from which it is made influence very greatly the number of β-ray recoil atoms that are collected. While this effect cannot be expected to apply to α-ray recoil atoms, since they penetrate deeply into the collecting surface, there is no definite evidence on the matter. The Faraday cylinder was therefore lined with lead, an isotope of the atoms of radium D which were to be collected.

A magnetic field of strength about 2000 gauss was applied perpendicular to the plane of the diagram. This was necessary to prevent secondary β-rays (δ-rays) which are produced at the source or screens in large numbers from reaching the Faraday cylinder. Similarly δ-rays produced inside the Faraday cylinder are prevented from getting out of it.

The current received by the Faraday cylinder was measured by a Dolezalek electrometer and Townsend induction balance. The electrometer and condenser were situated 2 metres from the source, to reduce ionization due to γ-rays as much as possible. Also the lead from the Faraday cylinder to the electrometer was taken through a brass tube, which was roughly evacuated in order to reduce electrical leaks caused by γ-ray ionization to a negligible amount.

The apparatus was exhausted by a mercury diffusion pump backed by a Fleuss pump, and the pressure could be reduced from atmospheric to ·0002 mm. in less than five minutes. A liquid-air trap between the pumps and gauges and the main recoil chamber removed mercury and other heavy vapours.

* *Zeit. f. Phys.* lix. p. 6 (1930).
† *C. R.* clxxxviii. pp. 1045, 1429.

The pressure was ordinarily measured by a MacLeod gauge, which always registered less than ·0001 mm. during a run; but to make quite certain that no vapours were present which would not be measured by the MacLeod gauge a quartz fibre vibration gauge was also used (see fig. 1). With this gauge a pressure of ·0002 mm. of any gas or vapour could be detected easily, and during a run it always showed less than this pressure.

Experimental Method.

The source to be examined was screwed into position, as shown in fig. 1, in front of the Faraday cylinder. The apparatus was then closed and exhausted, and when the pressure was sufficiently low observations were commenced. From the time the source was ready until observations began was about 10 minutes.

Observations were made of the charge received by the Faraday cylinder in unit time, with the windows (*a*), (*b*), and (*c*) successively rotated in front of the source. Each observation lasted from 20 to 60 seconds, and over such an interval of time, short compared with the life of the source, the strength of the latter may be considered constant. The mean time of each observation was noted, and the charge received per unit time by the cylinder was plotted against the time the source had been decaying. The smooth curves through the points corresponding to each of the windows (*a*), (*b*), and (*c*) are marked with these letters in the diagrams.

With window (*a*) in front of the source all particles with sufficient energy to pass through the magnetic field were received by the Faraday cylinder. With window (*b*) in front of the source the recoil atoms will be stopped by the collodion screen and only α- and β-rays received by the cylinder, while with window (*c*) only β-rays can reach it.

The difference of the ordinates of curves (*a*) and (*b*) gives the charge received by the Faraday cylinder per second due to recoil atoms, while the difference of curves (*b*) and (*c*) gives that due to α-particles. For every α-particle, of charge $+2e$, reaching the Faraday cylinder a recoil atom of unknown charge is also received, assuming the efficiency of recoil to be 100 per cent. This assumption should not be more than 20 per cent. from the truth provided the source is clean. Hence the average charge "Q" carried by a recoil atom can be compared with the charge on an α-particle.

It is to be noticed that, though the magnetic field employed (2000 gauss) was sufficient to prevent δ-rays or any β-particles

with energies under 10^4 electron-volts from reaching the Faraday cylinder, it is not strong enough to remove heavy ions of only a few volts energy from the beam of rays. The presence of such ions in considerable numbers was not suspected until later in the research, when special arrangements had to be made to remove them from the beam.

In these experiments it is not necessary to know the strength of the sources accurately, or in the case of a radium (B+C) source to know the relative amounts of radium B and radium C present, since the results depend only upon the differences in the ordinates of the curves. Nevertheless the source was always measured roughly at the beginning of a run, and the charge received by the Faraday cylinder compared with what would be expected from the strength of the source and the solid angle subtended at it by the opening in the Faraday cylinder. The agreement was satisfactory considering the approximate nature of the calculations.

Experimental Results.

The first experiments were made using sources prepared by exposing nickel disks in radon at a negative potential. Before use the disks were washed in methylated spirit and heated in a fairly good vacuum. The recoil atoms from these sources always appeared positively charged, and although Q was always constant throughout a run using one particular source it varied considerably for different sources.

As this method is open to the objection that the radium C may be partially buried in the metal surface, the well-known method due to von Lerch of depositing radium C on a nickel surface from a HCl solution was tried. In this way sources were obtained on a very clean polished nickel surface.

Most of the sources prepared in this way gave small positive values for Q, but for some it was almost zero and for a few it had a negative value. Most of the values found were fractional (*e.g.*, $+0{\cdot}3e$) and lay between $-1{\cdot}0e$ and $+1{\cdot}0e$. Also the charge on the recoil atoms appeared to increase, changing from negative to positive, when gas was admitted to the apparatus at such a pressure that the recoil atoms made collisions with gas atoms on their way to the Faraday cylinder. Thus curve "*a*" rose considerably, but curves "*b*" and "*c*" were unaffected when ·01 mm. of air was admitted to the chamber.

Efforts were next directed towards preparing the sources as cleanly as possible and under identical conditions each

time. These conditions seemed to be satisfied by the method of electrolysing radium C on to platinum from a solution of radium B + C in HCl*. The sources prepared in this way were even more inconsistent in their behaviour, the value of Q varying from $-1{\cdot}25e$ to $+4{\cdot}0e$ for different sources. In searching for the cause of the inconsistency in the results it was found that if a source on a nickel surface which was giving recoil atoms of apparent positive charge was removed from the apparatus and heated in air so as to oxidise the surface slightly and then returned to the apparatus the charge was invariably decreased, frequently becoming negative. In one instance Q dropped from $+2{\cdot}0e$ to $-1{\cdot}0e$, and was quite steady at that value until gas was admitted, when it increased with increasing pressure to about its original value. This effect will be discussed later.

These preliminary experiments led to the conclusion that the charge carried by a recoil atom was very strongly influenced by some factor which was not controlled in these experiments. After a search for the cause of the inconsistency of the results, it was traced to the presence of ions, both positive and negative, in the beam of rays collected by the Faraday cylinder. It was found that the positive charge received by the Faraday cylinder was greatly increased by raising the potential of the source to a few volts positive when the open window was in front of the opening in the Faraday cylinder. Also curve "*a*" dropped far below curve "*b*" when the source was kept at a negative potential of a few volts. Curves "*b*" and "*c*" were quite unaffected by alterations in the potential of the source, which is to be expected, since the ions have not sufficient energy to pass through even the thin collodion window.

The origin of these ions will be discussed later, but it is sufficient to say here that they cannot be produced by collisions of the recoil atoms with atoms of residual gas, since the pressure was always kept below 10^{-4} mm., and not more than 1 per cent. of the recoil atoms can then make collisions in their path to the Faraday cylinder.

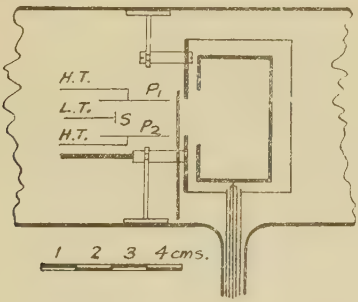
In order to remove these ions from the beam of rays the apparatus was remodelled as shown in fig. 2. The source S was fixed between two parallel horizontal nickel plates P_1 and P_2, 1 cm. apart, between which a strong electric field could be maintained. The radium C was deposited on a thin foil (3 mm. × 10 mm.) of nickel or platinum, and was held in position by heavy copper leads, which also served to carry a

* *Cf.* Rutherford, Chadwick, and Ellis, 'Radiations from Radioactive Substances,' p. 454.

current to heat the foil electrically. These current leads, as well as the high-tension leads to the parallel plates, were sealed in through the brass plate P (fig. 1). The position of the foil on which the source was deposited could be varied, but in the first experiments it was about 1·5 cm. from the end of the plates and 2·5 cm. from the opening in the Faraday cylinder. With these dimensions an ion will be captured by the plates when its energy, expressed in electron-volts, is approximately equal to the voltage between the plates. By moving the foil farther away from the Faraday cylinder ions of higher energy could be removed from the beam of rays.

By passing a current of about 6 amp. through the foil it could be raised to a red heat. Barton * has shown that raising a platinum foil on which radium C was deposited to

Fig. 2.



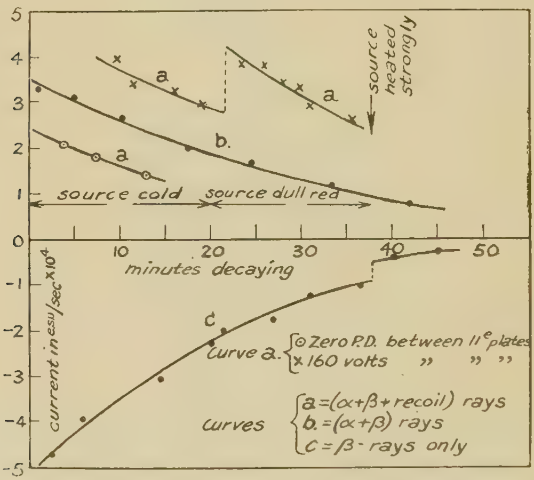
a visible red heat for a few seconds increased the efficiency of β-ray recoil. This increase is probably due to the removal of surface layers of adsorbed gases and vapours which impede the low energy β-ray recoil atoms. Hence in these experiments it seemed probable that such partial outgassing would decrease the chance of a recoil atom making a collision with an atom of adsorbed gas before it left the surface. The sources cannot be outgassed thoroughly, since radium C in the metallic state begins to volatilize at temperatures below 400° C. † When the radium C is in the form of an oxide it does not begin to volatilize until it is raised to about 700° C. In these experiments the radium C was apparently almost all in the metallic state, as considerable losses of the source were noticed when it was kept at a temperature judged by eye as about 550° C. for more than a few seconds.

* *Loc. cit.*
† St. Meyer u. E. Schweidler, 'Radioactivität,' p. 424.

The sources were prepared by electrolysis in the same way as described in the earlier experiments, and the surfaces of the foils always appeared quite clean after preparation. The source was kept in distilled water until it was quickly dried on filter-paper and introduced into the apparatus.

Each source was first examined cold, with and without a field between the parallel plates. Without the electric field the values found for the charge on the recoil atoms were quite as erratic as in the earlier experiments, but the application of a field of about 200 volts per cm. between the parallel plates immediately showed that Q had a positive

Fig. 3.



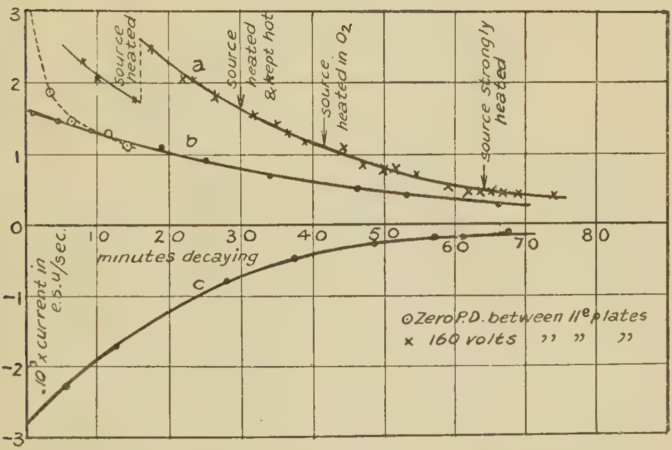
value of approximately $0{\cdot}5e$, which increased after heating the source to $1{\cdot}0e$.

The results of observations made with a radium C source deposited on a platinum foil are shown in fig. 3, the source being 14 mm. from the end of the parallel plates. During the first 20 minutes of the run observations were made without a field between the parallel plates (points shown thus ⊙) and with a field of 160 volts/cm. between them (points thus ×), when the open window "*a*" was in front of the source. These points were found to lie on curves respectively below and above curve "*b*," corresponding in the former case to a negative value of Q and in the latter to a positive value of about $+0{\cdot}4e$. As is to be expected, curves "*b*" and "*c*" are quite unaffected by this field. After observations had

been continued for 20 minutes the platinum foil was brought to a barely visible red heat by passing an electric current through it, and observations continued. The curve "a" has now risen much above curve "b," and the value of Q deduced from it is about $+1{\cdot}2\,e$. The distance between curves "b" and "c" falls off exponentially, with the half-value period of 19·5 minutes of radium C, within experimental error. The curve "a" also decays according to the same law.

After observations had been in progress for 38 minutes the source was heated to a bright red heat for 15 seconds and then cooled again. It was then found that all the curves had been considerably changed, due no doubt to loss and

Fig. 4.



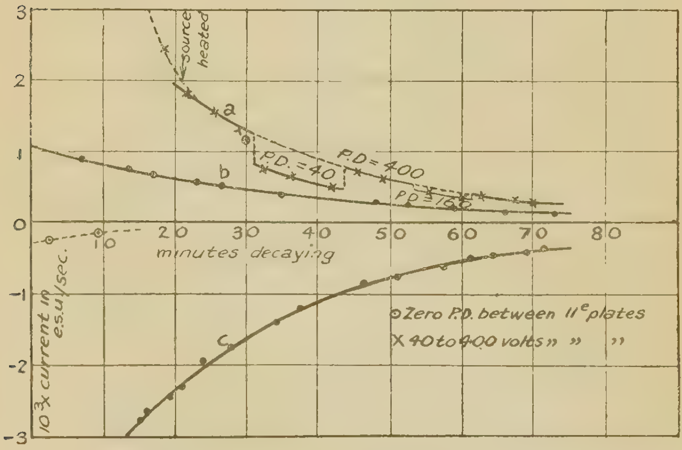
redistribution of the radium C, which would be volatilized at that temperature *in vacuo*.

In fig. 4 are shown the curves obtained with a similar radium C source on platinum. The general behaviour is much as before, except that the apparent value of Q without the field and from the cold source is almost zero. With 160 volts between the parallel plates it is found to be $+0{\cdot}5\,e$, and finally, after heating the source to a dull red glow for a few seconds and cooling again, it becomes very closely equal to $+1{\cdot}0\,e$. This value is not affected when the source is raised to a dull red heat (after 30 minutes decay), and kept constant at that temperature for 12 minutes. A pressure of 5 mm. of oxygen was then admitted, and the platinum foil raised to a bright red heat in it for half a minute and cooled. On pumping out the gas the value of Q was found to be

unchanged, and could not be changed by subsequent heating. Also no signs of volatilization were noticeable, the curves all maintaining their exponential form. This agrees with the view * that radium C oxide is less volatile than the element in the metallic state. It also indicates that the charge on the recoil atoms does not depend on whether the parent atom is a pure metal or an oxide.

In several runs, particularly when the source was only 1 cm. from the end of the parallel plates, and small fields, of the order of 200 volts, were applied between them, the value of Q found differed by as much as 40 per cent. from one positive unit. If, as should be the case, the efficiency of

Fig. 5.



recoil is nearly 100 per cent., the value of Q would be expected to be integral; but if high energy ions are present the variation of Q from unity may be due to those ions which have sufficient energy to avoid capture in the electric field and enter the Faraday cylinder.

The results shown in figs. 5 and 6 show that this is the case. In fig. 5 the curves show the result when the source is only 1 cm. from the end of the parallel plates and voltages of 40, 160, and 400 are applied between them. The observed points on curve "*a*" for each particular P.D. between the parallel plates are consistent amongst themselves and lie nicely on exponential decay curves, but increasing P.D. gives increasing values of Q.

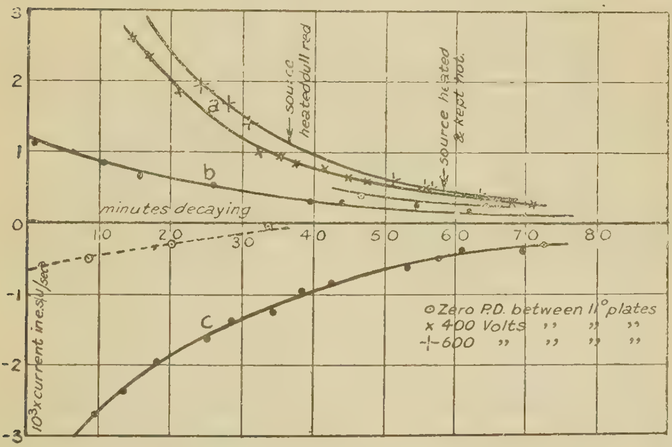
* Meyer u. Schweidler, 'Radioactivität,' p. 424.

Similarly for fig. 6, where the source is 14 mm. from the end of the parallel plates, there is a large increase in the value of Q when the P.D. is raised from 0 to 400 volts, and when it is raised to 600 volts Q becomes very closely equal to unity. Further increases in the distance of the source from the ends of the parallel plates up to 2 cm. and increases in the P.D. between them up to 800 volts gave no values of Q which were greater than $+1{\cdot}0\,e$ by more than the experimental error.

Thus, we have

	From fig. 5.			From fig. 6.		
P.D. in volts ...	40	160	400	0	400	600
Q	$\cdot35e$	$\cdot55e$	$\cdot75e$	$\cdot20e$	$\cdot70e$	$1{\cdot}0e$

Fig. 6.



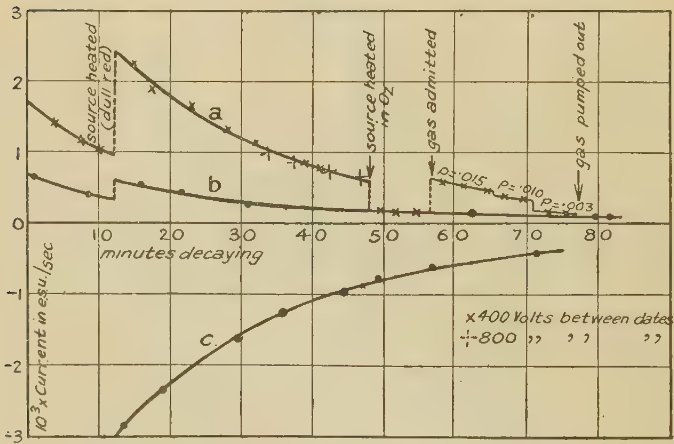
It is therefore concluded that the real value of the charge on a recoil atom coming from such a partially outgassed platinum surface *in vacuo* is one positive unit of electricity. The beam of recoil atoms is accompanied, however, by a stream of ions, both positive and negative, of which a number, comparable with the number of recoil atoms, have energies of the order of 1000 electron volts.

By applying a small positive accelerating potential to the source the apparent value of Q can be increased to five or ten times the true value if the source has not been baked out, and the order of double the true value after baking out. Similarly for a negative accelerating potential. This observation shows that the ions must be produced mainly in the

layer of adsorbed gas on the surface of the metal, which is at least partially removed by the heating *in vacuo*.

It is to be noticed in figs. 5 and 6 that, although Q appears negative in the early part of the run, when there is no field between the parallel plates, the value of Q found with a field of 400 volts between the plates is at first quite large, but drops quickly in a smooth curve to the value found after outgassing. In fact, the heating of the source has no effect. This is probably due to the fact that in these cases the platinum foil had been cleaned by heating strongly *in vacuo* before the radium C was deposited on it. In this way the

Fig. 7.



foil could be obtained in a much cleaner condition than by the normal methods of cleaning, with the result that it cleaned up of its own accord when subsequently placed in a vacuum.

It was mentioned, in describing the earlier experiments, that recoil atoms coming from a source on a slightly oxidized nickel surface appeared to have negative charges. This effect seemed so definite that it was investigated again after the method of removing the ions had been developed, and it was found to be partially true. In fig. 7 are shown the results obtained using a radium C source on a very clean nickel foil. The source was 2 cm. from the end of the parallel plates, and it is not surprising that there is no increase in the value of Q when the voltage between them is raised from 400 to 800 volts. The first part of the run is quite similar

to the runs already described using platinum foils, but after 48 minutes the foil was heated strongly in a few millimetres pressure of air, so that its surface became coated with a thin oxide film. As shown in the figure the curve "a" then drops and coincides with curve "b." This may be due either to the recoil atoms under these circumstances leaving the surface with no charge, or they may not leave the surface at all because of the oxidation. The latter hypothesis is shown to be untrue by the observation that when a small pressure of air of ·015 mm. was admitted to the apparatus the curve "a" immediately rose again as shown in fig. 7, giving an apparent value of $Q = +1{\cdot}85e$. As the gas was then pumped out in several stages, the curve fell in steps as the pressure decreased, until when the original pressure of ·001 mm. was attained Q again appeared to be zero. This rise in the curve "a" can only be due to positive charges produced when the recoil atoms collide with air atoms, and hence is strong evidence that the recoil atoms do leave the oxidized nickel foil. They must therefore be neutral in high vacuum when they originate at such a source. In no case did the recoil atoms appear to carry negative charges, as was found in the earlier experiments.

The fact that no similar effect was found when the source was heated on a platinum foil shows that it is due to oxidation of the metal surface itself rather than to oxidation of the radium C.

Discussion.

It has been shown that the average charge carried by a recoil atom of radium D originating from a source of radium C on a clean nickel or platinum surface is, in a good vacuum, one positive unit.

This conclusion is in agreement with the findings of Makower *, but not with the results of Wertenstein's † experiments. It seems possible that the high speed ions observed in these experiments were also present in those of Wertenstein, and, by chance, in such numbers as to give an apparent zero value for the charge on the recoil atoms.

The value found for Q of $+1{\cdot}0e$ is only a statistical average, and it must be borne in mind that a certain percentage of doubly charged and neutral recoil atoms may be present in the beam, though the close approximation of the value found to unity indicates that that is the normal charge of a recoil atom under the circumstances of these experiments.

* *Loc. cit.* † *Loc. cit.*

The experiments of Mund, Capron, and Jodogne* cannot be directly compared with those described here, since they investigated the charge on recoil atoms of radium A originating from radon, and these belong to different groups in the periodic table from the elements radium D and radium C respectively. Nevertheless their conclusion that the radium A atom is doubly ionized at the moment of its formation can be shown to be consistent with the present observation that a radium D recoil atom leaves a clean metal surface in a singly ionized state by considering the effect of the work function of the surface on the charge.

When the recoil atoms originated from a slightly oxidized nickel surface they were found to be neutral (see fig. 7). Such an effect would be expected if the work function of the surface was lowered by oxidation. The effect on the work function of a nickel surface of a layer of oxygen is not known, but it seems probable that the work function will be raised, as is the case with most metals. However, Oliphant † found that a gas-covered nickel surface, when bombarded by positive ions, emitted electrons much more freely than a surface which had been outgassed, and it was found in the writer's experiments that the proportion of negative ions escaping from a surface due to the action of the recoil atoms was greatly increased when it was slightly oxidized. These facts indicate that the recoil atoms, if positively charged initially, have a better chance of becoming neutral when they originate from an oxidized nickel surface, though the effect cannot be attributed solely to the influence of the work function.

If the radium D recoil atoms are really doubly ionized at the instant of their formation from radium C, as seems to be the case in the formation of radium A from radon, it seems certain that only a negligible percentage of them would escape from the surface without first extracting one electron from it, and so reducing their charge to one positive unit. The double ionization potential of radium D would probably be of the order of 15 to 20 volts, while the work function of a nickel or platinum surface is about 5 volts. Hence the chance of a doubly ionized recoil atom capturing one electron would be very great. If, however, it extracted one electron from the surface its electron affinity would be immediately lowered to about the same as that of the surface, and the probability of complete neutralization would be small.

* *Loc. cit.*

† Proc. Roy. Soc. A, cxxvii. p. 373 (1930).

Hence it appears that if the recoil atoms were initially doubly or singly charged in these experiments they would have left the surface of a clean metal with an average charge of one positive unit. The fact that they leave an oxidized nickel surface with no charge is also in agreement with one other experiment on an analogous problem.

One of the chief difficulties encountered in these experiments arose from the fact that large numbers of high energy ions, both positive and negative, are given off from the surface on which the active matter was deposited. These ions travel out from the surface of the source with sufficient energy to pass through the magnetic field (2000 gauss) and enter the Faraday cylinder. In the absence of the collodion window the charge received by the Faraday cylinder was greatly increased or decreased by the application of a small accelerating potential of two volts positive or negative respectively. From consideration of the energy required by an ion in order to pass through the magnetic field it was concluded that the greater number of these ions were oxygen or nitrogen ions with less than ten electron-volts energy. They were removed from the beam of rays by the application of a transverse electric field, and from the magnitude of the field required to finally eliminate their effects it was concluded that a considerable fraction of them must have energies as high as 500 electron-volts, and some even as high as 1000 electron-volts. Negative ions seemed to predominate in numbers at the higher energies.

These ions must be produced at the surface of the source by the action of the recoil atoms. There is definite evidence that they are not produced by the α-rays, since if this were the case we would expect similar ions to be liberated from the collodion window (*b*) on the side facing the Faraday cylinder from which the α-rays emerge. These ions would enter the Faraday cylinder and produce irregularities similar to those observed when no screen was placed in the path of the rays. No such irregularities were observed. Further, it is theoretically impossible for an α-particle to communicate such a large amount of energy to an atom except in very close collisions with the nucleus, which are so rare that they can be neglected in these experiments. Also, in the experiments to determine the number of α-rays emitted by radium * by collecting the α-rays in a Faraday cylinder and measuring the total charge received, the source is always covered by a thin screen sufficient to stop the recoil atoms. If charged

* Braddick and Cave, Proc. Roy. Soc. A cxxi. 1928, pp. 367 etc.

ions were set free from this screen by the α-particles, results entirely in disagreement with those of the many other methods, which could not be affected by such ions, would be expected. No such disagreement has been found.

There are two possible modes of production of the high speed ions. They may be produced by the action of the recoil atoms, which are shot into the metal surface with great energy, the process being analogous to the "sputtering" which takes place when a surface is bombarded with canal rays. It is difficult, however, to see how such sputtered particles could receive such large energies, for under ordinary conditions they are found to have energies of the order of one electron-volt. It seems much more probable that the ions are produced by the recoil atoms as they pass through the layer of adsorbed gas which certainly exists on the surface of the source before it is heated. It is to be expected that a recoil atom in passing through these adsorbed atoms would produce considerable ionization, and that in a few "head-on" collisions a large amount of energy would be transferred to the ions. This hypothesis is supported by the fact that the number of ions produced is greatly reduced by heating the source to about 500° C., which is sufficient to drive off most of the adsorbed gas. That some remain after heating is not surprising, since in no case was the heating sufficient to drive off the last monomolecular layer of oxygen which is known to persist on metal surfaces until the temperature is raised to 800° C. or more. It was not feasible to raise the temperature of the source sufficiently high to drive off the adsorbed gas completely on account of the volatilization of the radium C, which became serious at temperatures much over 500° C.

If a recoil atom of radium D, which has an initial energy of $1{\cdot}46 \times 10^5$ electron-volts, makes an elastic collision with a light gas atom (*e.g.*, H_2, O_2, N_2), the latter will recoil with an energy depending on the angle θ that the direction of its path makes with the original direction of flight of the recoil atom. The energies for different values of θ in the cases of hydrogen and oxygen are shown in the following table:—

θ / gas.	0°.	45°.	60°.	80°.
Hydrogen	2,800	1,400	700	—
Oxygen	45,000	22,500	11,250	1,350

From the tabulated figures it appears that if the ions observed are really oxygen ions they must be ejected at a large angle (greater than 80°) with the direction of motion of the recoil atom. Even allowing for a large dissipation of energy in the collision, the energy of a recoiling oxygen ion for which $\theta < 60°$ would still be far too great for it to be captured by the transverse electric field employed in these experiments. The calculated values for the energies of the recoiling hydrogen ions are more of the right order of magnitude to agree with the experimental results, but it is found that most of the high speed ions are negatively charged, and this seems definitely to exclude the possibility that they are hydrogen ions.

The most probable explanation that can be advanced for the origin of these ions is that they are produced by recoil atoms travelling nearly parallel with the surface of the source, and that they leave the surface in directions making angles of between about 10° and zero with the normal to the surface. The ions of higher energy which would make greater angles with the normal could not enter the Faraday cylinder. It is known that a recoil atom in passing through a gas travels in a practically straight path. Hence in most collisions the recoil atom suffers only a very small deflexion and the struck gas atom recoils in a direction making an angle of nearly 90° with its path. The relative number of gas ions whose paths make a small angle with the direction of motion of the recoil atom must be small, decreasing rapidly as the angle decreases from 90° to zero. These considerations account for the large number of low-energy ions observed, and also for the absence of very high-energy ions corresponding to "head-on" collisions with recoil atoms. If many such ions were produced by the recoil atoms which travel normal to the surface of the source they could not be stopped by the transverse electric field, and would enter the Faraday cylinder. Hence saturation of the ion current between the parallel plates could not be obtained at a potential difference of about 600 volts, as was observed.

Some knowledge of the behaviour of recoil atoms which escape from a surface which has not been outgassed can be obtained from these experiments, and it is of considerable importance since sources are seldom outgassed in radioactive experiments.

In figs. 3 and 4 it will be seen that the value of Q before outgassing was about $+0 \cdot 5\, e$. The same value for Q was found in almost all runs before outgassing, with the exception of those in which the foil had been thoroughly outgassed

in vacuo previous to the deposition of the radium C on it. The results then obtained are shown in figs. 5 and 6. It is improbable that the adsorbed layer of gas would decrease the efficiency of recoil by more than about 20 per cent. Hence the decrease in the charge received by the Faraday cylinder may be explained by assuming either that only 50 per cent. of the recoil atoms leave the surface charged or that negative ions are produced with sufficient energy to avoid capture in the transverse electric field. Since the recoil atoms must make many collisions in passing through this layer of adsorbed gas it is to be expected that they would emerge with a charge of at least $+2{\cdot}0\,e$. This expectation is based on the apparent increase of Q to several positive units when sufficient gas is admitted to the apparatus, the increase being proportional to the pressure. However, it is clear that a recoil atom would propel some of the gas atoms with which it collides in a direction making small angles with its own direction of flight. Most of these atoms would probably be ionized in the collision, and thus the increase in the charge received by the Faraday cylinder may be due to positive gas ions which are driven into it by the recoil atoms which themselves do not lose more than one or two electrons.

The apparent small value of Q before the source is outgassed may now be explained by supposing a process of capture and loss of electrons by the recoil atoms to take place in their passage through the gas layer, the net result of which is that only half of them emerge positively charged, the rest being neutral. An analogous process is known to occur in the passage of an α-particle through a gas, becoming very marked towards the end of its range, where its velocity is small. A similar effect also occurs during the passage of a positive ion through the gas in a discharge-tube. This explanation, however, does not seem to agree with the results obtained by Briggs* and Dee†, who found by different methods that 80 per cent. of recoil atoms still carried one unit positive charge at the end of their recoil path in air.

A more probable explanation seems to be that the recoil atoms in passing out through the layer of adsorbed gas are able to make a considerable number of "head-on" or very close collisions with atoms of gas when travelling in a direction nearly normal to the surface. The ions so formed would have sufficient energy to avoid capture by the transverse electric field and enter the Faraday cylinder, thus masking the true charge of the recoil atoms. Also it was

* Briggs, Phil. Mag. xli. p. 357 (1921).
† Dee, Proc. Roy. Soc. no. 116, p. 664 (1927).

found that negatively charged ions predominated in numbers amongst those of large energy, so we might reasonably expect those which are not captured to be mainly negative, which is in accord with the results found.

In the experiments, the results of which are shown in figs. 5 and 6, the value of Q is initially very large, and falls off rapidly to the normal value of $+1{\cdot}0\,e$, so that the heating has then no further effect. This is probably due to the very clean outgassed metal surface losing the layers of adsorbed gas much more rapidly *in vacuo* than a roughly cleaned foil. However, it is rather surprising that such a big difference in behaviour should exist.

In conclusion, it may be said that the value of the charge carried by recoil atoms of radium D determined in these experiments is probably the true value under the circumstances in which the atoms were investigated. It cannot, however, be regarded as giving the initial charge on the recoil atoms before they have interacted with a surface, since they originate from a surface. When allowance is made for the probable effect of the surface forces on the charge the results are shown to be in fair agreement with those of Mund, Capron, and Jodogne, in whose experiments this effect was avoided.

Summary.

The charge carried by α-ray recoil atoms of radium D escaping from a source of radium C has been investigated, and found to be one positive unit when the source is deposited on a clean nickel or platinum surface.

Evidence has been obtained that the charge carried by a recoil atom is influenced by interaction with the surface from which it escapes, and this theory has been extended to correlate the results of other observers with those obtained in these experiments.

The cause of earlier conflicting results has been traced to the presence of high-energy ions. A method of removing these from the beam has been developed, and a possible mechanism for their production suggested. The small average value of the apparent charge carried by recoil atoms before the source was outgassed is also explained by the presence of ions.

In conclusion, I wish to thank Professor Lord Rutherford for his continued interest in the work, and Dr. Chadwick, who suggested the research and gave much valuable advice and criticism during its progress. I also wish to acknowledge my indebtedness to Mr. G. R. Crowe for much assistance in the preparation of the active sources.

II. *Third-Order Terms in the Theory of the Stark Effect.*

By M. A. El-Sherbini, *B.Sc., The Faculty of Science, The Egyptian University, Cairo* *.

§ 1. *Introductory.*

THE effect of an electric field on the emission of spectral lines was first examined by J. Stark † in 1913. A first-order theory was given on the old quantum dynamics by K. Schwarzschild ‡ and P. Epstein § independently in 1916. The second-order terms, based on the old quantum theory, were worked out by P. Epstein ‖ and A. M. Mosharrafa ¶ independently. On the new wave mechanics Schrödinger ** has worked out first-order terms, and Epstein †† has obtained both first- and second-order terms. The second order terms have also been worked out simultaneously by Wentzel and Waller ‡‡, without using Schrödinger's perturbation theory.

The aim of this paper is to obtain third-order terms on the new theory. Although the effect of these terms is of little or no experimental significance at present, yet the work may be said to be of some theoretical interest; for each of the two previous approximations is characterized by the appearance of a new mode of vibration corresponding to the loss of a degree of degeneracy. When it comes to the third-order approximation, however, the system being already non-degenerate, no new frequency should appear. We find this is actually the case (see equation (19) below).

Another point of some theoretical interest is that the third-, like the first-order terms, affect the spectral lines symmetrically. This is due to the appearance of the factor $(m-n)$ in the expression for the increment of energy (see equation (21) below), which results in the occurrence of a negative value for the energy increment (and therefore for the line displacement) corresponding and numerically equal to every positive value.

* Communicated by Prof. A. M. Mosharrafa, Ph.D., D.Sc.

† *Berliner Sitzungsber.* Nov. 1913; *Ann. d. Phys.* xliii. pp. 965 and 983 (1914).

‡ "Zur Quantentheorie," *Berliner Sitzungsber.*, April 1916.

§ "Zur Theorie des Starkeffektes," *Ann. d. Phys.* l. p. 498 (1916).

‖ *Ann. d. Phys.* li. p. 184 (1916).

¶ Phil. Mag., Aug. 1922 and Nov. 1923.

** *Ann. d. Phys.* (4) lxxx. (1926).

†† Phys. Rev. Oct. 1926.

‡‡ G. Wentzel, *Zeit. f. Phys.* xxxviii. p. 518 (1927); J. Waller, *ibid.* p. 635.

§ 2. *Previous Work.*

The starting point is Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{2\mu}{k^2}(\mathrm{E}-\mathrm{U})\psi=0, \quad . \quad (1)$$

where μ and e are the mass and charge of an electron,

$$\left.\begin{aligned} k=\frac{h}{2\pi},\quad \mathrm{E}&=\text{the total energy,} \\ \mathrm{U}=\text{potential energy}&=\frac{-\mathrm{Z}e^2}{r}+e\mathrm{D}z \end{aligned}\right\} \quad . \quad . \quad (2)$$

$+\mathrm{Z}e=$ the charge on the nucleus, and $\mathrm{D}=$ the strength of the field.

Let the z-axis be in the direction of the field and use parabolic coordinates

$$\left.\begin{matrix} x=\sqrt{\xi\eta}\cos\phi, & y=\sqrt{\xi\eta}\sin\phi, & z=\dfrac{\xi-\eta}{2}, \\ 0\leqq\xi\leqq\infty, & 0\leqq\eta\leqq\infty, & 0\leqq\phi\leqq 2\pi. \end{matrix}\right\} \quad (3)$$

Equation (1) then becomes

$$\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\psi}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\psi}{\partial\eta}\right)+\frac{1}{4}\left(\frac{1}{\xi}+\frac{1}{\eta}\right)\frac{\partial^2\psi}{\partial\phi^2}+\frac{\mu}{2k^2}$$

$$\left[\mathrm{E}(\xi+\eta)+2\mathrm{Z}e^2-\frac{e\mathrm{D}(\xi^2-\eta^2)}{2}\right]\psi=0\,. \quad . \quad . \quad (4)$$

Make the substitution

$$\psi=\mathrm{M}(\xi)\mathrm{N}(\eta)\,{\cos \atop \sin}\,(s-1)\phi.$$

We get for the functions M, N the two ordinary differential equations

$$\left.\begin{aligned} \frac{d^2\mathrm{M}}{d\xi^2}+\frac{1}{\xi}\frac{d\mathrm{M}}{d\xi}+\left(\frac{\mu\mathrm{E}}{2k^2}+\frac{\mathrm{A}}{\xi}-\frac{(s-1)^2}{4\xi^2}-\frac{\mu e\mathrm{D}}{4k^2}\xi\right)\mathrm{M}=0, \\ \frac{d^2\mathrm{N}}{d\eta^2}+\frac{1}{\eta}\frac{d\mathrm{N}}{d\eta}+\left(\frac{\mu\mathrm{E}}{2k^2}+\frac{\mathrm{A}'}{\eta}-\frac{(s-1)^2}{4\eta^2}+\frac{\mu e\mathrm{D}}{4k^2}\eta\right)\mathrm{N}=0. \end{aligned}\right\} \quad . \quad . \quad . \quad (5)$$

where $$\mathrm{A}+\mathrm{A}'=\frac{\mu\mathrm{Z}e^2}{k^2}.$$

Simplify the equations by substituting

$$M=\xi^{(s-1)/2}e^{\alpha\xi}X(\xi), \quad N=\eta^{(s-1)/2}e^{\alpha\eta}Y(\eta),$$

$$\alpha=\sqrt{-\frac{\mu E}{2k^2}}. \quad \ldots \quad (6)$$

Equations (5) are reduced to

$$\left.\begin{aligned}\frac{d^2X}{d\xi^2}+\left(2\alpha+\frac{s}{\xi}\right)\frac{dX}{d\xi}+\left[\frac{A+s\alpha}{\xi}-\frac{\mu eD}{4k^2}\xi\right]X=0,\\ \frac{d^2Y}{d\eta^2}+\left(2\alpha+\frac{s}{\eta}\right)\frac{dY}{d\eta}+\left[\frac{A'+s\alpha}{\eta}+\frac{\mu eD}{4k^2}\eta\right]Y=0.\end{aligned}\right\} \quad (7)$$

Let $\quad A_0+s\alpha_0=-2\alpha_0 m, \qquad A_0'+s\alpha_0=-2\alpha_0 n\,; \quad . \quad (8)$

$$\therefore \qquad \alpha_0=-\frac{\mu Ze^2}{2k^2(m+n+s)} \quad \ldots \quad (9)$$

and

$$E_0=-\frac{2k^2}{\mu}\alpha_0^2=-\frac{\mu Z^2e^4}{2k^2(m+n+s)^2}. \quad \ldots \quad (10)$$

§ 3. *Third-Order Terms.*

We solve by the method of successive approximation. We set

$$\left.\begin{aligned}\alpha&=\alpha_0+D\alpha_1+D^2\alpha_2+D^3\alpha_3+\ldots,\\ A&=A_0+DA_1+D^2A_2+D^3A_3+\ldots,\\ A'&=A_0'-DA_1-D^2A_2-D^3A_3-\ldots,\\ X&=X_0+DX_1+D^2X_2+D^3X_3+\ldots,\\ Y&=Y_0+DY_1+D^2Y_2+D^3Y_3+\ldots,\\ E&=E_0+\Delta_1E+\Delta_2E+\Delta_3E+\ldots.\end{aligned}\right\} \quad \ldots \quad (11)$$

Substitute in equation (7), we get for X,

$$\frac{d^2X_0}{d\xi^2}+\left(2\alpha_0+\frac{s}{\xi}\right)\frac{dX_0}{d\xi}-\frac{2\alpha_0 m}{\xi}X_0=0, \quad \ldots \quad (12)$$

$$\frac{d^2X_1}{d\xi}+\left(2\alpha_0+\frac{s}{\xi}\right)\frac{dX_1}{d\xi}-\frac{2\alpha_0 m}{\xi}X_1$$

$$=-2\alpha_1\frac{dX_0}{d\xi}-\frac{A_1+s\alpha_1}{\xi}X_0+\frac{\mu e}{4k^2}\xi X_0, \quad \ldots \quad (13)$$

$$\frac{d^2X_2}{d\xi^2}+\left(2\alpha_0+\frac{s}{\xi}\right)\frac{dX_2}{d\xi}-\frac{2\alpha_0 m}{\xi}X_2$$
$$=-2\alpha_1\frac{dX_1}{d\xi}-\frac{A_1+s\alpha_1}{\xi}X_1+\frac{\mu e}{4k^2}\xi X_1$$
$$-2\alpha_2\frac{dX_0}{d\xi}-\frac{A_2+s\alpha_2}{\xi}X_0, \quad . \quad (14)$$

$$\frac{d^2X_3}{d\xi^2}+\left(2\alpha_0+\frac{s}{\xi}\right)\frac{dX_3}{d\xi}-\frac{2\alpha_0 m}{\xi}X_3$$
$$=-2\alpha_1\frac{dX_2}{d\xi}-\frac{A_1+s\alpha_1}{\xi}X_2+\frac{\mu e}{4k^2}\xi X_2$$
$$-2\alpha_2\frac{dX_1}{d\xi}-\frac{A_2+s\alpha_2}{\xi}X_1-2\alpha_3\frac{dX_0}{d\xi}-\frac{A_3+s\alpha_3}{\xi}X_0, \quad (15)$$

with four corresponding equations for Y.

I have evaluated X_2 from equation (14) as a sum of functions of the form $CX_0(m', s)$, and X_1 has already been expressed by Epstein * in the same form. Therefore equation (15) can be reduced to the form

$$\frac{d^2u}{d\xi}+\left(2\alpha_0+\frac{s}{\xi}\right)\frac{du}{d\xi}-\frac{2\alpha_0 m}{\xi}u=\frac{CX_0(m', s)}{\xi}, \quad . \quad (16)$$

whose solution † is

$$u=\frac{C}{2\alpha_0(m'-m)}X_0(m', s). \quad . \quad . \quad . \quad (17)$$

This solution satisfies all the requirement of finiteness when $m' \neq m$. So the condition which we have to impose on the third-order terms in equation (15) is that the sum of the coefficients of $X_0(m, s)$ must vanish.

Proceeding in this manner I have obtained ‡

$$(2m+s)\alpha_3+A_3$$
$$=-\frac{20\mu^2e^2\alpha_1}{64\times 32k^4\alpha_0^6}(s+2m)(34m^2+34sm+4s^2+9s+5)$$
$$+\frac{12\mu e\alpha_1^2}{64k^2\alpha_0^4}(6m^2+6ms+s^2+s)$$
$$-\frac{\mu e\alpha_2}{8k^2\alpha_0^3}(6m^2+6ms+s^2+s)$$

* Phys. Rev. xxviii. no. 4, October 1926, p. 700, equation (28).

† *Ibid.* p. 700, equation (26).

‡ The full details are somewhat cumbersome and have been omitted here for brevity.

$$+\frac{8\mu^3e^3}{32\times32\times64k^6\alpha_0{}^8}(750m^4+1500m^3s+996m^2s^2$$
$$+258m^2s+330m^2+246ms^3+258ms^2$$
$$+330ms+16s^4+59s^3+73s^2+30s), \quad . \quad . \quad . \quad (18)$$

and a corresponding condition (obtained from the equations for Y), which may be derived from (18) by interchanging m and n and reversing the signs of A_3 and μ respectively. From these two conditions we obtain by addition and substitution for α_1 and α_2 in terms of m, n, and s (see Epstein's paper quoted above, equations (29) and (30), pages 701 and 702),

$$\alpha_3=\frac{3\mu^3e^3(m-n)}{128\times64k^6\alpha_0{}^8}$$
$$\{3(m+n+s)^2+10(m-n)^2+10s^2-20s+20\}, \quad (19)$$

or since

$$\Delta_3E=-\frac{2k^2}{\mu}(\alpha_0\alpha_3+2\alpha_1\alpha_2)D^3, \quad . \quad . \quad . \quad (20)$$

we finally have

$$\Delta_3E=-\frac{3\mu^2e^3D^3(m-n)}{64\times64k^4\alpha_0{}^7}$$
$$\{20(m+n+s)^2-11(m-n)^2+s^2-2s+30\}$$
$$=\frac{3D^3}{32\mu^5Z^7\times e^{11}}\times\left(\frac{h}{2\pi}\right)^{10}(m-n)(m+n+s)^7$$
$$\{20(m+n+s)^2-11(m-n)^2+s^2-2s+30\}. \quad (21)$$

My thanks are due to Prof. A. M. Mosharrafa, under whose direction the above work was done.

III. *The Existence of the J-Phenomena.* By Ivor Backhurst, *M.Sc., Physics Department, National Physical Laboratory, Teddington, Middlesex* *.

Introduction.

In association with the scattering of X-rays, phenomena have been observed which were at first (8, 9) attributed to the excitation of "J" characteristic radiation of an irradiated atom, but were later (11, 12) thought to be

* Communicated by Dr. G. W. C. Kaye, O.B.E.

due to the occurrence of a "J-transformation" of the scattered radiation during its passage through an absorbing medium. It has been supposed [20] that the appearance or non-appearance of these phenomena was determined by some unknown factor, since apparently similar experimental conditions prevailed in either case. For some years experiments on "J-transformations" have, from time to time, been described[11] to [30], although in the same period a number of investigators have reported failure to find any evidence for such phenomena[31] to [44]. In 1929, Barkla and Sen Gupta [29] described an experiment in which heterogeneous radiation from a Coolidge tube was scattered at 90° by a slab of paraffin wax placed at 45° to the incident radiation. One ionization chamber S was placed to receive the scattered beam and another T to receive the transmitted beam. Sheets of aluminium of total thickness x were placed in the path of the scattered beam, and a number of sheets of the same total thickness were placed in the path of the transmitted beam. The ratio of the ionization current measured by S to that measured by T was plotted for different values of x. The discontinuous nature of the curve obtained was said to be due to sudden changes of intensity of the partially absorbed scattered beam. The changes were of the order of 7 per cent., and were supposed to be due to "J-transformations."

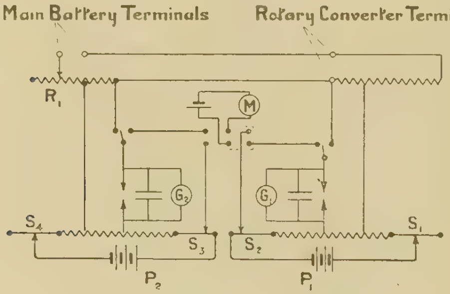
The extent to which it is justifiable to regard such a curve as discontinuous would naturally depend on the amount of fluctuation due to experimental error. Unsteadiness of the X-ray source would normally be thought the chief cause of error, but Barkla, to whom we owe the conception of J-phenomena, has emphasised the importance of maintaining steady conditions of X-ray intensity in order to obtain the phenomena. On this account it seemed improbable that the changes of intensity of 7 per cent., referred to above, could be accounted for in this way. Barkla has, however, suggested the desirability of investigating these phenomena with the aid of a "constant potential" high tension generator, and an attempt to do this is described below.

Apparatus.

The "constant potential" high tension plant used had full wave rectification, and has been described by E. Bell[1].

It was supplied with alternating current from a rotary converter driven by accumulators. The input voltage to the converter was maintained within a range of $\pm 0{\cdot}2$ per cent. of its mean value by means of a potentiometer arrangement P_1 (fig. 1) and a hand operated rheostat R_1 in the supply circuit. The current supplied to the converter was held within $\pm 0{\cdot}5$ per cent. of its mean value by means of another potentiometer arrangement P_2 (fig. 1) and a hand operated rheostat in the circuit supplying current to the filament of the X-ray tube. All the potentiometer resistances were eureka and, with the exception of the slide wires S, variable only in fixed steps. No sliding contacts were used; the contacts on the slide wires were screwed, and all

Fig. 1.

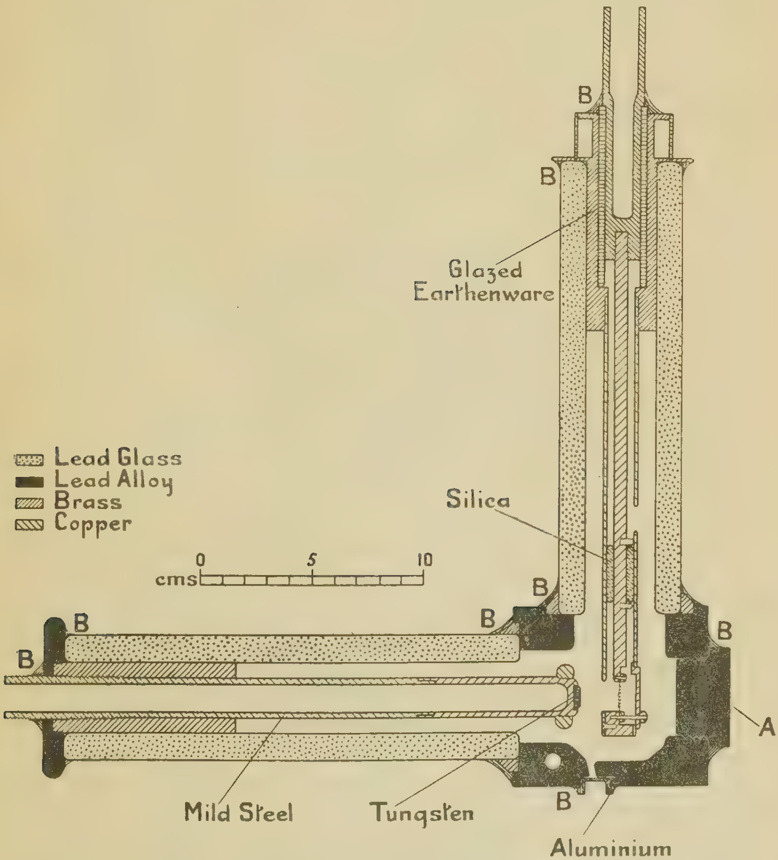


other contacts were screwed or soldered. The cadmium cell balance was checked at frequent intervals r with the aid of a microammeter (M), one division deflection of which corresponded to a change of potentiometer battery voltage of 0·03 per cent. The galvanometers employed in the voltage and current measuring potentiometers had time periods of 2 and 0·5 seconds respectively, and were not far from aperiodic under the conditions obtaining. A change of 1 per cent. in the voltage or current produced a deflexion of 11 or 6 divisions respectively in the corresponding galvanometers G_1 and G_2.

A rheostat was included in the circuit between converter and high tension transformer, but this was used only when putting the plant into operation, and was brought to zero resistance when taking readings in order that current fluctuations should affect the high tension voltage as little as possible. Change of voltage was effected

by an auto-transformer. The high tension voltage was continuously indicated by an attracted disk electrometer, the calibration of which was verified by comparison with a 10 cm. sphere spark gap. The mid-point of the secondary winding of the high tension transformer was earthed,

Fig. 2.



and for part of the investigation sufficient voltage was obtained by arranging the constant potential plant so that one output lead was earthed. When this was done the potentiometer P_2 was used to measure the high tension current directly, instead of the converter current as shown in the diagram.

The X-ray tube (fig. 2) was constructed specially for

research on X-ray scattering. The objects aimed at in particular were (*a*) high power for continuous operation, (*b*) exceptionally high X-ray protection, and, (*c*) the facilitation of large angle scattering measurements. The importance of (*a*) and (*b*) is considerable, since the intensity of scattered radiation is in general only a small fraction of that of the primary beam. The main features of the X-ray tube are as follows.

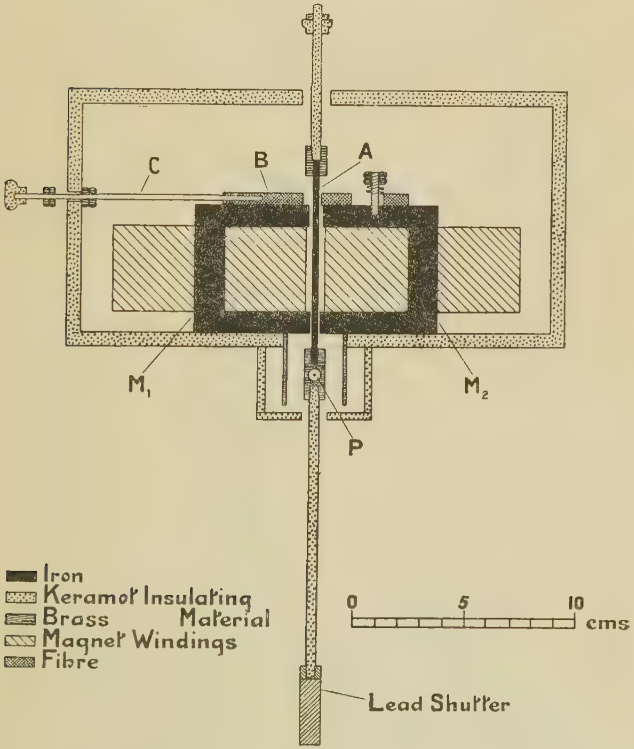
The anticathode stem and cathode are contained in lead glass tubes emerging at right angles from a lead alloy box of minimum wall thickness 1 cm. and at earth potential. The tungsten anticathode is situated in the lead alloy box 4 cm. from an aluminium window. The glass tubes have a lead equivalent of 3 mm., but since the radiation from the anticathode is everywhere incident obliquely on the glass the minimum effective stopping power is equivalent to 5 mm. of lead. A spectrometer may be brought close to the aluminium window, and undue restriction of the maximum scattering angle measurable is avoided.

The anticathode is cooled by a forced circulation of oil, which passes through ebonite tubes to a water cooler at earth potential. Thermo-syphon water cooling is used for the cathode, and the lead alloy box is water cooled from the main. Removal of the plug A permits the replacement of a filament to be effected without difficulty. A vacuum is maintained by means of oil diffusion pumps. It was found that for outputs exceeding 1·3 kilowatts the lead glass surrounding the anticathode stem became too much heated. The capacity of the X-ray tube was, however, found ample for the purpose of the investigation.

The X-ray tube was continuously in operation, while readings were being taken, X-ray exposure times of multiples of half a minute being obtained by means of a specially constructed exposure meter (fig. 3). The meter was controlled by a master clock and in order to maintain the accuracy of the meter as high as that of the clock it was arranged to operate without the aid of a mechanical relay. Some form of relay action had to be introduced, however, as the electrical pulse available from the clock circuit was of too low power and short duration to actuate with sufficient speed and certainty a lead shutter of the requisite dimensions. In the arrangement adopted, a lead shutter 3 mm. thick is

connected by a keramot rod, pivoted at the centre of gravity, to an iron strip A lying between the poles of two electromagnets. The main windings of the magnets are connected in series and included in a Laboratory clock circuit. A rotary switch short-circuits either or both of these windings. Auxiliary windings on the two magnets are supplied with unequal continuous currents

Fig. 3.



through a change over switch. The iron strip A is pushed clear of the pole-face of the weaker magnet, say M_2, by the slotted fibre bar B, the auxiliary currents being adjusted so that the resultant magnetic force on A is just sufficient to hold it in contact with B under the obtaining conditions of vibration.

During a half-minute interval between consecutive current pulses from the master clock the rotary switch is set to allow the following impulse to increase the pull

of M_1 and thereby supply a "trigger" action resulting in the movement of the lead shutter out of the path of the X-ray beam.

Succeeding current pulses produce no effect on the meter until the currents through the auxiliary windings are interchanged, and the rotary switch and fibre bar actuated, allowing the following impulse to terminate the exposure. The auxiliary currents are necessary to prevent the iron strip A rebounding from the magnet pole-faces and also serve to accelerate greatly the motion of the shutter. The above device was found completely

TABLE I.

Stopwatch observations.		Calculated equal time intervals.		Differences.
Mins.	Secs.	Mins.	Secs.	Secs.
0	5·82	0	5·820	0
0	35·79	0	35·788	−0·002
1	5·76	1	5·756	−0·004
1	35·72	1	35·724	0·004
2	5·69	2	5·692	0·002
2	35·66	2	35·660	0·000
3	5·60	3	5·628	0·028
3	35·58	3	35·596	0·016
4	5·55	4	5·564	0·014
4	35·53	4	35·532	0·002
5	5·49	5	5·500	0·010
5	35·47	5	35·468	−0·002
6	5·46	6	5·436	−0·024
6	35·42	6	35·404	−0·016
7	5·37	7	5·372	0·002
7	35·34	7	35·340	0

reliable and enabled exposures of half-a-minute to be automatically timed to within 0·1 per cent. The accuracy of the meter was checked by observing stopwatch readings without stopping the watch while listening to the "ticks" made by the meter as the iron strip A hit the magnet polefaces. Some readings obtained are shown in Table I.; the bulk of the error indicated is clearly observational, and not due to the meter, but the maximum total error is within the estimate given above.

The X-ray intensity-time product was measured by means of an air-filled ionization chamber and Compton electrometer. In order to have a short time period and stable zero, the electrometer was used with a short

quartz fibre and adjusted to a low sensitivity (1000 mm. per metre per volt). It was used as a null instrument, the insulated system being coupled through a small air-condenser to a potentiometer constructed to read to thousandths of its full range. The values of the condenser and potentiometer current were adjusted so that one small potentiometer unit corresponded approximately to 1 mm. per metre deflexion of the electrometer needle.

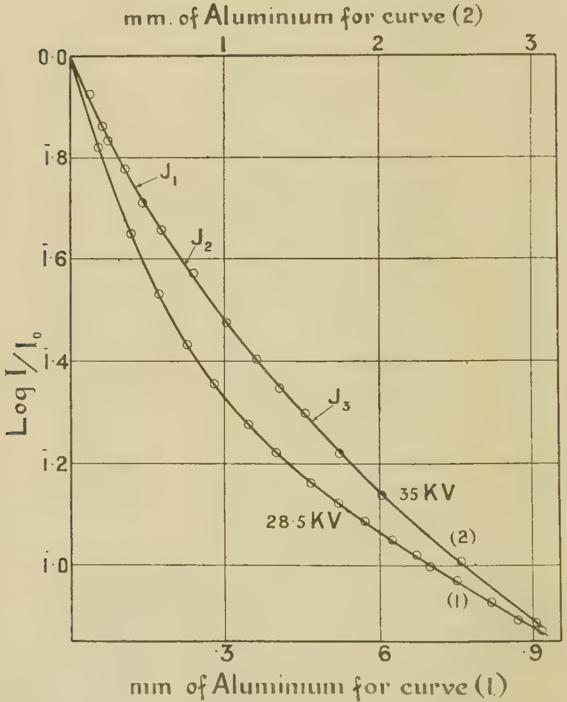
Method of Experiment and Results.

When an attempt was made to repeat the experiment described by Barkla and Sen Gupta it was found that the arrangements described above secured sufficient constancy of X-ray intensity to render unnecessary comparison with the transmitted beam. The experiment consisted, therefore, in measuring the absorption in aluminium of the beam scattered at 90°. The scattering material used was a slab of paraffin wax placed so that a normal to the face of the slab was the external bisector of the angle of scattering. An area approximately 2 mm. wide and 2 cm. high of the face of the slab was irradiated, the angular divergence of the incident beam in the plane of the scattering angle being about 1° 4′. In the same plane the angular divergence of the scattered beam received by the ionization chamber was 1° 48′. The solid angle subtended at any point in the aluminium absorber by the effective aperture of the ionization chamber was less than 0·0027 of 4π, so that the absorption measured was the total absorption, *i. e.*, only a negligible fraction of the radiation scattered by the aluminium absorber could enter the ionization chamber.

High tension voltages were chosen to obtain radiation of the right "hardness," as measured by a "half-value" thickness of aluminium, to cover the region in which the J-phenomena have been found. In particular J-discontinuities have been stated to occur [24] for values of μ/ρ in aluminium of 3·76, 3·24, 2·44, 1·94, 1·40, 0·73, 0·47 corresponding to half-value thicknesses of 0·683, 0·792, 1·051, 1·322, 1·833, 3·52, 5·46 mm. respectively. The first six of these positions are marked on the curves shown in figs. 4 and 5. For curve (1) the X-ray tube window was a sheet of aluminium 0·03 mm. thick, while for the other curves it was 0·5 mm. thick. The curves

have been scaled to make the average gradient of each somewhere near 45° in order to make as marked as possible any deviations of the experimental points. In order that the extent of fluctuations due to experimental error may better be gauged, the observations from which curve (3) of fig. 5 is deduced are given in Table II. in the order in which they were obtained. The other curves

Fig. 4.

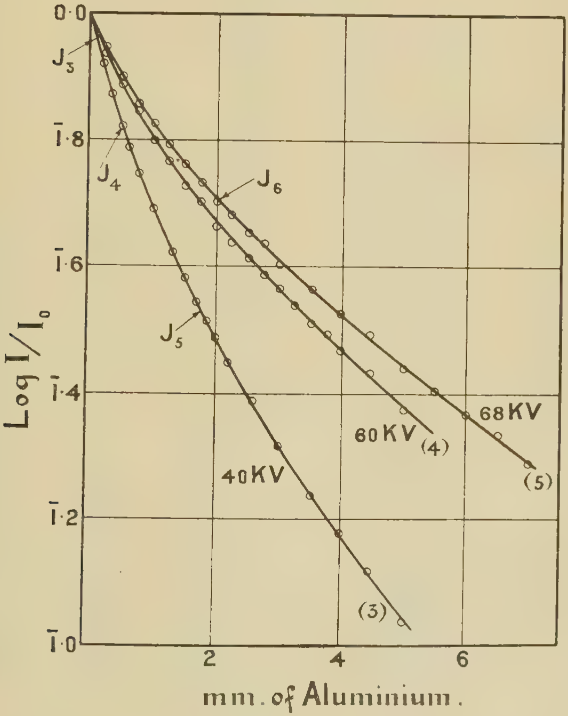


were deduced from similar sets of readings, each point being calculated from the average of two, or sometimes three, consecutive potentiometer readings.

Log (I/I_0), instead of I/I_0, has been plotted in order that the distance of a point from the curve, when measured along the ordinate, should be directly proportional to the percentage error in the ratio I/I_0. From measurements made in this way on large scale graphs, an analysis is given in Table III. of the extent of the departure of the

experimental points from a smooth curve drawn so that $f''(x)$ is always positive. The result is evidently consistent with the assumption that absorption of a heterogeneous beam is determined by the intensity distribution over its constituent wave-lengths in the manner given by the equation $I=\int I_\lambda e^{-\mu_\lambda x} d\lambda$ where $I_\lambda \, d\lambda$ is the initial intensity

Fig. 5.



associated with the wave-length range $d\lambda$ at the wave-length λ, μ_λ is the corresponding absorption coefficient and I is the total intensity of the transmitted radiation. This is mentioned as the validity of the above assumption appears to have been questioned [11, 23].

It will be noticed that the average percentage error shown in Table III. is several times the estimated error in the constancy of the supply current and voltage. Although this is partly due to the fact that a perfectly

constant input to a high tension plant would not necessarily mean a perfectly constant output, it is mainly due to the proportionality factor existing between change of high tension voltage and change of intensity of the scattered radiation. Thus $\Delta I/I = k \cdot \Delta V/V$ where k is always greater than unity and becomes greater the higher the

TABLE II.

Values for curve (3) *of fig.* 5.

MM. of Aluminium.	Potentiometer Readings.	A Averages.	B Inter-polated values for 0 mm.	log (A/B) =log (I/I_0).
0	966·5, 972	969·2	—	—
2·009	298, 297	297·5	968·6	$\bar{1}$·4874
3·017	200, 200	200	968·0	$\bar{1}$·3152
4·000	145, 145·5	145·2	967·4	$\bar{1}$·1764
0	968·5, 965	966·8	—	—
0·207	804, 805·5	804·8	966·9	$\bar{1}$·9203
0·351	721, 721	721	,,	$\bar{1}$·8726
0·514	642, 640	641	,,	$\bar{1}$·8215
0·636	595, 594	594·5	,,	$\bar{1}$·7888
0·797	542, 541	541·5	,,	$\bar{1}$·7482
1·008	475, 477	476	,,	$\bar{1}$·6922
1·337	405, 406·5	405·8	,,	$\bar{1}$·6229
0	967, 967	967	—	—
1·522	368, 368	368	967·1	$\bar{1}$·5804
1·705	339, 339	339	967·3	$\bar{1}$·5446
1·868	314·5, 315	314·7	967·4	$\bar{1}$·5124
2·216	273, 270, 271	271·1	967·6	$\bar{1}$·4473
2·600	236, 235·5	235·7	967·7	$\bar{1}$·3867
3·531	165, 168, 168	167	967·9	$\bar{1}$·2369
0	969, 967	968	—	—
4·470	126, 126·5	126·7	966	$\bar{1}$·1163
5·010	104, 106	105	964	$\bar{1}$·0371
2·009	297, 296	296·5	962	$\bar{1}$·4889
0	961, 959	960	—	—

absorption of the scattered beam. The value of k under various conditions was determined by changing the high tension voltage about 2 per cent. In Table IV. are given values of k corresponding to conditions of " hardness " and absorption under which the J-phenomena have mostly been found. A 1 per cent. change in supply current was found to produce a 1 per cent. change, or less, in the intensity of the scattered beam for any of the high tension voltages employed.

Taking the factor k into account, the percentage errors shown in Table III. correspond to the estimated variation in voltage and current. For example, the

TABLE III.

Point number on curve.	Percentage departure from curve of the ratio I/I_0. Curve Number.				
	1	2	3	4	5
1	0·0	1·2	0·0	0·2	0·0
2	0·0	0·7	0·0	0·2	0·2
3	0·0	0·2	−0·1	0·5	−0·5
4	0·0	−0·2	0·0	−0·5	0·7
5	0·0	0·0	0·0	0·4	0·9
6	0·0	0·5	−0·6	0·3	0·3
7	0·0	0·5	0·4	1·6	0·4
8	0·0	−0·9	0·0	−0·9	−0·5
9	0·0	0·7	0·3	−0·5	−0·9
10	0·2	0·0	−0·2	0·3	0·0
11	−0·3	0·2	0·4	−0·9	1·2
12	0·8	−0·5	−0·9	0·5	−1·5
13	−0·7	−1·2	0·6	0·2	0·0
14	0·3	0·6	−0·5	−0·6	0·1
15	0·0	−0·9	−0·6	0·9	1·6
16	−0·6	—	0·8	−0·5	0·8
17	0·8	—	1·1	1·2	0·4
18	—	—	−1·9	−1·4	0·0
19	—	—	—	—	1·4
20	—	—	—	—	−1·0

TABLE IV.

Kilovoltage.	MM. of Aluminium.	K=percentage intensity increase for 1 per cent. increase in kilovoltage.
36	0	3·04
	0·348	5·1
	0·815	5·9
43	0	1·7
	1·522	3·4
	3·017	4·9
54	4·001	4·9
	6·018	5·5

maximum error of −1·9 per cent. (Point no. 18 of Curve 3) occurs when the ratio I/I_0 is approximately 0·1, and a unit of the electrometer potentiometer is therefore 1 per cent. of I/I_0. Actually, however, the reading was

estimated to 0·5 per cent. of I/I_0. The factor k was over 6, so that an effective voltage variation of (1·9–0·5)/6 or 0·23 per cent. would account for this discrepancy without considering effects due to current supply variation or variations occurring in the high tension plant. Clearly, therefore, there is no room for the supposition that J-discontinuities of a magnitude approaching 7 per cent. exist under the conditions obtaining in these experiments.

Critical Summary and Discussion of Evidence for J-Phenomena.

Prior to about 1923, the theory of the existence of J-phenomena depended on experiments of a different character from those considered above. To make the position clear, these early experiments are briefly summarized and criticised below.

(1) Barkla (6, 8) found that the ratio of absorption in iron to absorption in aluminium, when plotted against wavelength, gave a curve showing a sharp change in slope at $\lambda=0·5$ Å.U. and concluded that this was occasioned by a selective J-absorption in aluminium.

(2) Williams (3) found that the curve obtained by plotting absorption in aluminium against absorption in copper showed a discontinuity at $\lambda=0·49$ Å.U. and attributed it to J-absorption in aluminium.

Richtmyer (34) in 1921 found these results could not be confirmed and Duane and Shimizu (31) had previously shown that no characteristic radiation of aluminium existed in this region of wave-length.

(3) Barkla and Miss White (7) measured the absorption in aluminium, paper, water, paraffin wax, and copper, of filtered primary rays. Curves of relative absorption, obtained by plotting for different " effective " wavelengths the mass absorption coefficients of the first four materials against the mass absorption coefficient of copper, showed discontinuities thought to indicate J selective absorption.

As pointed out by Richtmyer (35), the data for these curves showed discontinuities at points other than those cited as positions of J-absorption edges, and the experimental accuracy was clearly insufficient to warrant the conclusions drawn.

(4) Owen (2) noticed a small subsidiary minimum in the spectral intensity curve of radiation from a palladium

anticathode and suggested that it might possibly be occasioned by a selective $J\beta$ absorption in the silicon of the carborundum analysing crystal.

Siegbahn and Wingardh [32] pointed out that the minimum could have been occasioned by selective K-absorption in the palladium anticathode.

(5) Dauvillier [4] obtained spectral intensity curves of primary rays. When aluminium filters were used he detected an irregularity at 0·358 Å.U. which he attributed to J-absorption in aluminium. Richtmyer [35] suggested this might be due to K-absorption of the iodine in the ionization chamber. As Dauvillier remarked at the time, however, this irregularity was practically within the limits of experimental error.

Dauvillier also noticed a small peak in the tungsten spectrum which he attributed to J-absorption in the bromine of the ionization chamber, but, as Richtmyer pointed out, this was presumably the $K\alpha$ peak of tungsten.

(6) Crowther [5] measured the absorption of rays scattered from aluminium placed in the path of a heterogeneous beam. He found the scattered rays more heterogeneous than the primary, comprising longer wave-lengths than the latter, and attributed this to J characteristic radiation from the aluminium scatterer. Alternatively, he considered it possible that the primary radiation might suffer a slight increase in wave-length in the process of scattering. Numerous experiments by Compton and others have shown the latter hypothesis to be the correct one.

As far as the writer is aware, no evidence for J-phenomena in addition to the above has at any time been brought forward except by Barkla and his co-workers [9–29], who have carried out experiments in which the essential feature has been the detection of "J-transformation" discontinuities in curves showing the relative intensities of transmitted and scattered rays after partial absorption by various equal thicknesses of some material. Some slight irregularities in early spectroscopic data have been considered by Khastgir and Watson [13, 14] to support the "J-transformation" theory, but, as pointed out by Siegbahn [39] and by Nipper [40], later spectroscopic measurements with improved apparatus have not shown these irregularities.

The position is, therefore, that all the early results thought to show J-phenomena can either be attributed to experimental error or else explained in terms of phenomena, the existence of which has been well confirmed. The hypothesis of J-phenomena must therefore depend entirely on the validity of the later results [9–29] concerning "J-transformations" of scattered radiation, in which the discontinuities have been stated to be 7·5 or 10 per cent., and which have been performed with a knowledge of criticisms passed on the earlier work and of the failure of a number of investigators to obtain corroborant results. Summaries of the published accounts of experiments on "J-transformations" have been given by Gaertner [42] and by Alexander [44]. These investigators and also Dunbar [38, 43] and Worsnop [41] have endeavoured without success to obtain J-phenomena. Gaertner comments on the lack of information concerning arrangements made to secure accuracy in the J-experiments. Dunbar concludes that J-irregularities may have been due to varying amounts of soft radiation reaching the measuring electroscopes, and considers that the method of experiment was unsuitable to distinguish between the effects of classical and modified scattering. Worsnop remarks that some unknown condition must be necessary for the occurrence of J-phenomena. Alexander concludes that the factors necessary for such are restricted to very special experimental conditions. A brief account of J-phenomena is given by Barkla in the International Critical Tables, and reference to later unpublished work has recently been made by him [30].

The conclusion reached by the writer is that "J-transformations" have mainly been due to the following causes:—

(1) Fluctuations, both irregular and periodic, in the voltage-time curve of the high tension generator.

(2) Change of wave-length in the process of scattering, taking into account the effect of twice scattered radiation.

(3) A systematic tendency on the part of the investigators concerned to overestimate the sensitivity and accuracy of the balance method of measurement they used.

(4) The vitiation of results by entry into the ionization chambers of "stray" scattered radiation from surrounding objects.

The object of the balance method used throughout in the J-experiments has been to minimise error due to fluctuations of intensity of the beam incident on the scattering material. The intensity ratio of scattered to transmitted radiation can, however, remain constant only if the intensity of the incident beam varies in such a manner as to maintain unchanged the relative intensities of its component wave-lengths. Otherwise, variation in the intensity ratio must necessarily occur, since the scattering coefficient is not independent of wave-length, and a considerable variation must result from the dependence on wave-length of the proportion of modified to unmodified scattering. A change in the high tension voltage-time curve will therefore, in general, alter the intensity ratio, although the peak voltage may remain unchanged. The latter case may be regarded as a change in the peak voltage of part of the X-ray beam, and this change will be magnified by different amounts in its effect on the intensities of the partially absorbed transmitted and scattered beams, since the *k* factors for these beams will not be the same. In this connexon it may be observed that

(*a*) by gradually changing the frequency of interruption of the primary current to the induction coil the J-discontinuities could be made to become less sharp and finally disappear [23];

(*b*) when a high tension transformer was used, together with a Coolidge tube, the J-discontinuities were found to be under "perfect control" [24];

(*c*) the condition for and character of J-discontinuities has been considered to be dependent on some unknown factor in the method of excitation of the X-rays [23, 28].

It is impossible to believe that the Compton effect can have failed to be present to an observable extent in most of the J-experiments of which there are published accounts. The intensity ratio of scattered to primary rays in these experiments was almost always found to decrease with increasing absorption of each beam by equal thicknesses of material, the decrease occurring either

evenly or in steps. Since the ratio was found to be constant between adjacent steps it was deduced that the Compton effect was not operative. On the other hand, no evidence was produced that the accuracy of any of these experiments was sufficiently high to show with certainty the change of ratio to be expected from the Compton effect in the interval between consecutive steps. In some J-experiments [21] in which the range of wave-length was stated to be from 0·3 to 0·6 Å.U. it was calculated that the Compton effect should have produced a total change in absorption coefficient of from 12 to 20 per cent., of which one-tenth could easily be measured. The conclusion reached was that there was no evidence at all of the existence of the Compton effect.

In actual fact it can be shown that with scattering materials of very low atomic weight such as those used in the J-experiments and with incident radiation of wave-length in the neighbourhood of 0·3 Å.U. the observed change of absorption coefficient is greater than that calculated by means of the equation $\lambda'-\lambda=0{\cdot}024\,(1-\cos\phi)$ even if it be assumed that the whole of the scattered radiation is modified. By altering the size of the scatterer it may be proved that a large part, if not quite all, of this extra change in absorption is due to the effect of radiation twice scattered and modified in the scatterer. Using a crystal selected incident beam, the writer has measured the intensity of the scattered beam (*a*) without an absorbing screen, (*b*) with an absorbing screen in the incident beam (*c*) with the same absorbing screen in the scattered beam. Values found for a scattering angle of 90° with paraffin wax as scattering material and an absorbing screen of copper $0{\cdot}210\mp0{\cdot}005$ mm. thick are shown in Table V.

The calculated increase in absorption coefficient is obtained using the equation $\mu=3{\cdot}34+1{,}340\lambda^3$ (sufficiently exact in the neighbourhood of 0·3 Å.U.) to calculate λ from μ and then using the derived expression

$$(\mu'-\mu)/\mu=\left(1-\frac{3{\cdot}34}{\mu}\right)\left\{\left(\frac{\lambda'}{\lambda}\right)^3-1\right\}$$

Similar results have also been obtained with beryllium or water as scattering materials, the range of scattering angle investigated being from 20° to 120° *. It should

* It is hoped to publish later a full account of scattering experiments with a crystal selected incident beam.

be mentioned that the cross-section of the incident beam at the surface of the paraffin slab was 0·5 mm. by 16 mm., the angular divergence being 11 minutes of arc. The cross-section, at the slab, of the scattered beam reaching the ionisation chamber was 5 mm. by 16 mm. For scattered beams of larger cross-section the increase in absorption coefficient becomes greater.

The effect of heterogeneity of beam is to make the apparent change of wave-length, as deduced from the absorption coefficients, somewhat less than the true value. In the J-experiments [21], however, in which it was

TABLE V.

Scatterer.		Intensity Readings.	Interpolated value for (*a*)	Intensity Ratio.	Linear Absorption Coefficient (μ).	Experimental increase in μ.	Calculated increase in μ.
Paraffin wax cylinder	*a*	708·5					
Diameter 3 mm...	*b*	278·5	708·3	2·543	44·4		
Height 25 mm. ..	*c*	224·5	708·1	3·154	54·7	23·1%	22·9%
	a	708					
Paraffin wax slab	*a*	991					
	b	391	990·3	2·533	44·3		
Thickness 15 mm.	*c*	297	989·7	3·332	57·3	29·5%	
	a	989					

concluded that the Compton effect did not exist, the main factor in reducing the apparent change of wave-length would be the proportion present of unmodified long wave-lengths, the existence of which would render any calculation of wave-length change from absorption measurements quite valueless. It appears impossible to escape the conclusion that the sensitivity of J-experiments of this type, in the estimation of change of quality of scattered radiation, has been considerably overestimated by the investigators concerned.

Other results found in connexion with J-experiments seem to be capable of easy explanation. In particular, it has been stated that radiation scattered from thin

sheets of material, *e. g.*, paper, was found to be unmodified, while that scattered from thick sheets was modified (28). In the latter case, since in general, large angle cones of radiation have apparently been used, a large proportion of twice scattered and modified radiation would enter the ionization chamber, and the apparent change in absorption coefficient would be considerably greater than in the former case, in which the intensity of the scattered radiation would necessarily be less and the consequent reduced accuracy of the experiment probably insufficient to show definitely the smaller amount of modification actually present.

An alteration of the absorption of one X-ray beam in a sheet of material, due to the transmission through the sheet, in another direction, of a second X-ray beam has been found in some of the J-experiments, *e. g.* (29). Some time ago the writer endeavoured to find some trace of such an effect without success. Pseudo effects of this character were, however, very easily obtained due to "stray" scattered radiation, the reduction of which to completely negligible proportions was found to require care.

It has been argued that, since some of the J-experiments showed J-phenomena while other apparently similar experiments did not, no negative result could be regarded as disproving the existence of the phenomena. On the other hand, whether this argument may be valid or not, a careful experimental investigation, together with a perusal of the literature, leads inevitably to the conclusion that up to the present time no experimental work has been reported that does, in fact, constitute any real evidence for the existence of J-phenomena, and there seems to be no reason of any kind why the existence of such phenomena need be postulated.

In conclusion I desire to express my thanks to Dr. G. W. C. Kaye, O.B.E., the Superintendent of the Physics Department, for his interest in this investigation, and to acknowledge efficient assistance rendered by Mr. P. R. Pallister, B.Sc. in the experimental work. The X-ray tube and X-ray exposure meter described were made by Mr. W. G. H. Turl in the Instrument Workshop of the Laboratory, the high lead content glass tubes having been specially manufactured due to the courtesy of Messrs. Philips of Holland.

Summary.

A full-wave rectification "constant potential" high tension generator has been employed in an attempt to obtain evidence of the existence of J-phenomena. The X-ray tube, of which a description is given, was specially designed for use in intensity measurements of scattered X-rays and adapted for excitation by a "mid-point earthed" high tension generator. Potentiometers were used in the control of supply voltage and current, and intensity measurements were made by means of a potentiometer coupled through a condenser to the insulated system of the ionization chamber and Compton electrometer. X-ray exposures were automatically timed by means of a simple type of exposure meter, operated without mechanical relay by a master clock.

With these arrangements, sufficiently accurate absorption curves of radiation scattered from paraffin wax at 90° were obtained, without recourse being necessary to the custom, in J-experiments, of making comparison measurements of the transmitted beam. No indication of J-phenomena was found, but the intensity of the scattered beam was found to be highly sensitive to change of high tension voltage. Values of the proportionality factor for small percentage changes were determined for conditions of hardness and absorption under which the J-phenomena have been stated to occur.

A short critical discussion of the literature is given, together with a bibliography. In particular, comment is made on the alleged non-appearance of the Compton effect in some of the J-experiments, and some results obtained with a crystal selected incident beam are given to show that the actual change of absorption is greater than the theoretical value on account of the effect of twice scattered radiation. It is considered that in all the reported accounts of J-transformations the investigators concerned have overestimated the accuracy of their experiments. Probable causes of error are enumerated, the chief being fluctuation of the time voltage curve of the high tension plant.

It is concluded that so far there has been no real experimental evidence whatever for J-phenomena, and that there seems to be no reason why the existence of such should be postulated.

References.

(1) Bell, Brit. Journ. of Rad. ii. p. 156 (1929).

EVIDENCE FOR J-PHENOMENA.

(2) Owen, Proc. Roy. Soc. A. xciv. p. 339 (1918).
(3) Williams, Proc. Roy. Soc. A. xciv. p. 567 (1918).
(4) Dauvillier, *Ann. de Phys.* xiii. p. 49 (1920).
(5) Crowther, Phil. Mag. xlii. p. 719 (1921).
(6) Barkla, Proc. Roy. Soc. A. xcii. p. 501 (1916).
(7) Barkla and Miss White, Phil. Mag. xxxiv. p. 270 (1917).
(8) Barkla, Phil. Trans. of the Roy. Soc. A. ccxvii. p. 315 (1918).
(9) Barkla, 'Nature,' cxii. p. 723 (1923).
(10) Barkla and Miss Sale, Phil. Mag. xlv. p. 737 (1923).
(11) Barkla, 'Nature,' cxiv. p. 753 (1924).
(12) Watson, Proc. Roy. Soc. of Edinburgh. xlv. p. 48 (1924–5).
(13) Khastgir and Watson, 'Nature,' cxv. p. 604 (1925).
(14) Khastgir and Watson, 'Nature,' cxvi. p. 47 (1925).
(15) Barkla and Miss Mackenzie, 'Nature,' cxvi. p. 942 (1925).
(16) Barkla and Khastgir, Phil. Mag. xlix. p. 251 (1925).
(17) Barkla, Phil. Mag. xlix. p. 1033 (1925).
(18) Barkla and Khastgir, Phil. Mag. l. p. 1115 (1925).
(19) Barkla and Khastgir, 'Nature,' cxvii. p. 228 (1926).
(20) Barkla, 'Nature,' cxvii. p. 448 (1926).
(21) Barkla and Miss Mackenzie, Phil. Mag. i. p. 542 (1926).
(22) Barkla and Khastgir, Phil. Mag. ii. p. 642 (1926).
(23) Barkla and Miss Mackenzie, Phil. Mag. ii. p. 1116 (1926).
(24) Barkla and Watson, Phil. Mag. ii. 1122 (1926).
(25) Barkla, 'Nature,' cxix. p. 778 (1927).
(26) Barkla and Khastgir, Phil. Mag. iv. p. 735 (1927).
(27) Watson, Phil. Mag. v. p. 1146 (1928).
(28) Barkla, Phil. Mag. v. p. 1164 (1928).
(29) Barkla and Sen Gupta, Phil. Mag. vii. p. 737 (1929).
(30) Barkla, 'International Critical Tables,' vi. p. 1 (1929) and 'Nature,' cxxvii. p. 877 (1931).

EVIDENCE AGAINST J-PHENOMENA.

(31) Duane and Shimizu, Phys. Rev. xiv. p. 389 (1919).
(32) Siegbahn and Wingardh, *Phys. Zeit.* xxi. p 83 (1920).
(33) Richtmyer and Grant, Phys. Rev. xv. p. 547 (1920).
(34) Richtmyer, Phys. Rev. xvii. p. 434 (1921).
(35) Richtmyer, Phys. Rev. xviii. p. 13 (1921).
(36) Richtmyer, Phys. Rev. xix. p. 418 (1922).
(37) Compton, 'Nature,' cxiii. p. 160 (1924).
(38) Dunbar, Phil. Mag. xlix. p. 210 (1925).
(39) Siegbahn, 'Nature,' cxvi. p. 11 (1925).
(40) Nipper, 'Nature,' cxvi. p. 12 (1925).
(41) Worsnop, Proc. Phys. Soc. xxxix. p. 305 (1927).
(42) Gaertner, *Phys. Zeit.* xxviii. p. 493 (1927).
(43) Dunbar, Phil. Mag. v. p. 962 (1928).
(44) Alexander, Proc. Phys. Soc. xlii. p. 82 (1930).

IV. *On the Elastic Extension of Metal Wires under Longitudinal Stress.*—Part II. *Experimental.*

By L. C. Tyte, *B.Sc., A.Inst.P., Research Department, Woolwich* *.

Introduction.

The object of the research was the measurement of deviations from Hooke's law, for which purpose the apparatus described in a previous paper [1] was devised.

The materials investigated were steel, iron, nickel, brass, copper, aluminium, zinc, tin, and lead, some of them also being examined after different heat treatments.

The Elastic After-effect.

It has long been known that, when stress is applied to a body, even within the "elastic limit," it suffers an immediate extension and a further small one which increases with time, the rate of increase slowly falling to zero. A similar effect has been observed in recovery from strain after the removal of stress, and has been called "Elastische Nachwirkung" or the elastic after-effect.

This phenomenon has been noted by previous investigators of the deviation from Hooke's law. Thompson [2] performed special experiments to determine its magnitude. He found on loading a wire, most of the extension occurred in the first 2 seconds, a small additional extension in the next 11 seconds, and an almost negligible increase in the next 17 seconds. His results for brass were :—

Preliminary tension.	Alteration in length		Total alteration.
	from 2–13 secs.	from 13–30 secs.	
kgm.	mm.	mm.	mm.
0·6	0·035	0·001	0·036
1·2	0·062	0·010	0·072
1·8	0·100	0·020	0·120

Further, he showed that the alteration occurring between 2–13 seconds could be considered as the heat expansion caused by loading.

* Communicated by Prof. C. H. Lees, D.Sc., F.R.S. Published by permission of the Ordnance Committee.

Similar effects have been obtained by Grüneisen [3] for lead, tin, zinc, and bismuth, and by Schülze [4] for steel and brass. The latter, however, concludes that the adiabatic elastic modulus (determined acoustically) depends only on the load and is independent of the magnitude and duration of the after-effect.

Finally, Kyrillov [5] measured the small elongations in steel wire 16 m. long, produced by constant surcharge ($\Delta P = 2$ kgm.) of loading with different initial loads ($P = 14$–66 kgm.). The elongations (2·22–2·24 mm.) were measured by an electrical method with an accuracy of 1/2000 mm. Although the wire was kept for a long time heavily loaded, the small surcharge produced gradual but very slow elongation—in the mean about 0·0002 mm. per minute. All the measurements were reduced to the same duration (3 minutes) of action of surcharge, and to the same temperature. In a note at the end of this paper Weinberg compares the values with those of Thompson, who found that Young's Modulus diminished by 4 per cent. for elongations which were less than half those employed by Kyrillov. In the latter's experiments variations of Young's Modulus did not exceed 1 per cent. when calculated from the initial length and cross-sectional area, and were within 0·8 per cent. if based on actual values of these quantities. The variation in Thompson's values of Young's Modulus was attributed to his having excluded the possibility of hardening (écrouissage) taking place by only loading for a few seconds.

Preparation of Specimens.

In order to obtain consistent results, great care had to be taken to ensure that the wires were free from all kinks and curves before the commencement of an experiment. This was done by electrically heating the wires suspended vertically with a load at their lower ends sufficient to keep them taut. This treatment was continued until the wires were free from all irregularities, when they were allowed to cool slowly by reducing the current in stages. All the specimens were prepared in this manner except a few lengths of steel and brass which were used without treatment.

Experimental Method.

After this treatment the experimental wires, of length 1 and 2 metres respectively, were mounted in the chucks and then supported in their vertical position with sufficient

load to keep them taut. (For further details see Part I.) The scale reading was observed for this initial load, and a given increment was added. This stretched the wires, and the excess extension of one of them caused a scale deflexion to be observed in the telescope. The reading, however, did not remain at this new value but slowly changed, indicating a progressive extension with time. This subsequent motion gradually decreased, finally falling to zero, the time required for this depending on the load applied and the hardness of the material—for tin or lead with loads exceeding the yield-point the increase of extension with time seemed to continue indefinitely. The magnitude of this subsequent extension was comparable with the initial extension, and in the case of the soft metals many times greater. The striking contrast is exhibited by the following typical observations for steel and tin; decrease of scale reading corresponds to increase of length.

Steel: Unheated Specimen 2.

Scale reading.	Time.		Scale reading.	Time.	
cm.	h.	m.	cm.	h.	m.
19·28 (11 kilos. applied) .	2	45	14·84 (16 kilos. applied).	10	55
18·85		50	12·12	11	0
18·83		56	12·08		6
18·83 (12 kilos. applied) .	3	6	12·07		15
18·25		10	12·07		22
18·25 (13 kilos applied) .		15			
17·58		24			
17·56		34			
17·53 (14 kilos applied) .		40			
16·7		43			
16·57		55			
16·55	4	5			

Tin: Preliminary Set. 206 gm. load on wires.

Scale reading.	Time.		Scale reading.	Time.	
cm.	h.	m.	cm.	h.	m.
21·18 (load applied) ...	1	32	20·24	3	13
20·98		38	20·12		27
20·88		47	20·08		33
20·82		53	19·80	4	9
20·66	2	13	19·38		50
20·66		20	18·98	5	50
20·53		36	18·52	6	48
20·44		45	18·38	7	15
20·30		58			

The relation between the increase of excess extension and time would naturally be a complicated function and no attempt has been made to connect them, but curves between the scale readings and time are given in figs. 1 and 2 for steel and tin.

Thus in the present experiments, owing to the greatly improved method of observing the excess extension, the elastic after-effect presented a serious problem. Instead of adopting the arbitrary method employed by Thompson and Kyrillov of taking the extension after a given time interval, the wires were allowed to complete their extension before the observation was taken. This gives results for a definite state, *i. e.* complete extension, covers the difficulty of hardening—this being done automatically for each stage of the loading—and also allows the wires to take up the air temperature, thus eliminating the heat expansion caused by loading.

A typical set of observations are given below for iron.

Iron: Specimen 3.

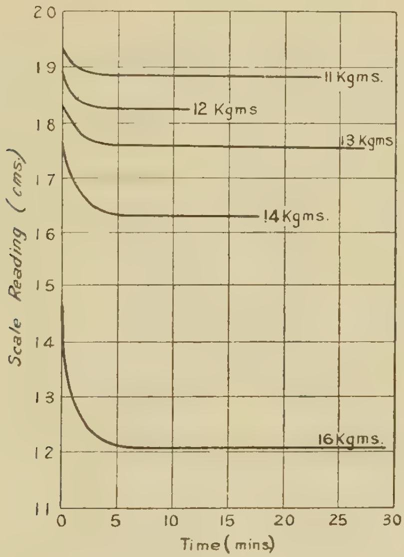
Load.	Scale reading.	Deflexion.	Temperature.	Time.	
gm.	cm.	cm.	°	h.	m.
1000	21·00	—	17·1 C.	11	50
2000	21·00	—	17·4	12	0
3000	20·97	0·03	17·5		10
4000	20·94	0·06	17·5		25
5000	20·91	0·09	17·6	1	0
6000	20·88	0·12	17·8		30
7000	20·82	0·18	17·7	2	5
8000	20·75	0·25	17·9		25
9000	20·68	0·32	17·8		50
10000	20·53	0·47	18·1	3	35
11000	20·42	0·58	17·6	4	0
12000	20·10	0·90	17·5		30

Experimental Results.

The observations have been summarized in the following tables, and are shown graphically in the corresponding figures. The load given in the tables does not include the initial load of chucks, ratio-bar, etc., but is the load applied through the ratio-bar to the two wires, and hence the load on the shorter is two-thirds and on the longer one-third

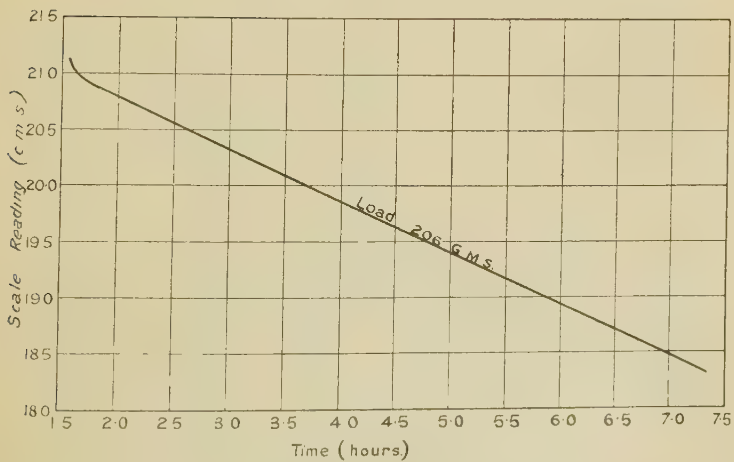
of this. Where a dash has been placed in the excess extension columns, it indicates that the corresponding load was necessary as an additional load to keep the wires taut.

Fig. 1.



Scale reading—time curves for steel.

Fig. 2.



Scale reading—time curve for tin.

STEEL.

No. 30 : s.w.g. bright drawn steel wire supplied by the London Electric Wire Company and Smith's ; a single wire would support a load of 16 kgms. without fracture.

Density = 7·639 gm./c.c. Initial load on short wire = 153 gm.

Constant of apparatus = 0·009308. " " long " = 69 "

1. *Heat Treatment* A. None ; wire used as supplied. Mean diameter = 0·304 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.).												Mean Values I.–X.	Actual excess extension of shorter wire. (10^{-3} cm.).
		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	V *a*.	VI *a*.		
1	9·2	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	18·4	0·03	0·02	0·03	0·04	0·03	0·02	0·03	0·02	—	—	—	—	0·02$_2$	0·20
3	27·6	0·06	0·05	0·06	0·11	0·07	0·06	0·10	0·08	0·03	0·03	0·01	—	0·06$_5$	0·60
4	36·7	0·12	0·12	0·15	0·18	0·13	0·13	0·21	0·19	0·09	0·12	0·04	0·02	0·14	1·34
5	45·9	0·25	0·24	0·24	0·32	0·25	0·23	0·36	0·36	0·20	0·24	0·11	0·08	0·27	2·50
6	55·1	0·38	0·36	0·40	0·51	0·42	0·38	0·59	0·56	0·33	0·41	0·17	0·19	0·43	4·04
7	64·3	0·56	0·54	0·55	0·72	0·64	0·56	0·81	0·82	0·51	0·65	0·25	0·31	0·64	5·92
8	73·5	0·78	0·76	0·75	1·00	0·89	0·84	1·09	1·12	0·75	0·94	0·33	0·43	0·89	8·30
9	82·7	1·02	1·02	1·02	1·31	1·22	1·15	1·41	1·48	1·04	1·24	0·45	0·56	1·19	11·09
10	91·8	1·32	1·31	1·34	1·75	1·58	1·51	1·76	1·91	1·36	1·62	0·57	0·70	1·55	14·39
11	101·0	1·71	1·70	1·77	2·25	2·00	1·95	2·17	2·39	1·78	2·08	0·73	0·89	1·98	18·44
12	110·2	2·17	2·20	2·30	2·90	2·53	2·54	2·70	2·95	2·32	2·65	0·92	1·16	2·53	23·51
13	119·4	2·76	2·85	2·94	3·58	3·16	3·21	3·32	3·62	2·99	3·41	1·15	1·50	3·18	29·64
14	128·6	3·50	3·80	3·80	4·49	3·92	4·06	4·06	4·49	3·78	4·10	1·45	1·96	4·00	37·24
15	137·8	4·46	5·33	5·01	5·62	4·92	5·22	5·00	5·50	4·95	5·15	1·82	2·51	5·12	47·62

Experiments V *a*. and VI *a*. were performed by reloading the wires used in experiments V. and VI., the maximum load on the shorter wire used in V. being 165·3 kgm./sq.mm. and in VI. 183·7 kgm./sq. mm.

2. *Heat Treatment* B. Wires heated by a current of 3 amps. and slowly cooled, *i. e.* to a temperature of 280° C. approximately.

Mean diameter = 0·303 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	Mean Values.	Actual excess extension of shorter wire. (10^{-3} cm.).
1	9·2	—	0·01	0·00	0·00	0·00	—
2	18·5	0·00	0·02	0·01	0·01	0·01	0·09
3	27·7	0·01	0·04	0·02	0·02	0·02	0·19
4	37·0	0·03	0·06	0·04	0·04	0·04	0·37
5	46·2	0·06	0·08	0·06	0·07	0·07	0·65
6	55·5	0·10	0·11	0·09	0·10	0·10	0·93
7	64·7	0·15	0·15	0·13	0·14	0·14	1·30
8	74·0	0·19	0·19	0·17	0·18	0·18	1·68
9	83·2	0·25	0·24	0·22	0·22	0·23	2·14
10	92·5	0·32	0·30	—	0·28	0·30	2·79
11	101·7	0·40	0·38	—	0·34	0·37	3·44
12	111·0	0·50	0·49	—	0·42	0·47	4·37
13	120·2	0·63	0·70	—	0·52	0·61	5·68

3. *Heat Treatment* C. Wires heated by a current of 3·6 amps. and slowly cooled, *i. e.* to a temperature of 400° C. approximately.

Mean diameter = 0·303 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	Mean Values.	Actual excess extension of shorter wire. (10^{-3} cm.).
1	9·2	—	—	—	—
2	18·5	0·00	0·01	$0·00_5$	0·05
3	27·7	0·02	0·03	$0·02_5$	0·23
4	37·0	0·05	0·06	$0·05_5$	0·51
5	46·2	0·09	0·10	$0·09_5$	0·88
6	55·5	0·14	0·14	0·14	1·30
7	64·7	0·20	0·20	0·20	1·86
8	74·0	0·27	0·27	0·27	2·51
9	83·2	0·36	0·37	$0·36_5$	3·40
10	92·5	0·45	0.49	0·47	4·37
11	101·7	0·60	0·63	$0·61_5$	5·72
12	110·0	0·81	0·84	$0·82_5$	7·68

4. *Heat Treatment* D. Wires heated by a current of 4 amps. and slowly cooled, *i. e.* to a temperature of 500° C. approximately.

Mean diameter =0·302 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
1 ……	9·3	—	—	—	—	—	—
2 ……	18·6	0·00	0·02	0·00	0·01	0·01	0·09
3 ……	27·9	0·02	0·05	0·02	0·03	0.03	0·28
4 ……	37·2	0·05	0·09	0·06	0·06	$0·06_5$	0·60
5 ……	46·6	0·09	0·14	0·11	0·11	0·11	1·24
6 ……	55·9	0·16	0·21	0·19	0·18	$0·18_5$	1·72
7 ……	65·2	0·26	0·29	0·29	0·27	0·28	2·59
8 ……	74·5	0·38	0·39	0·45	0·38	0·40	3·72
9 ……	83·8	0·59	0·51	0·70	0·51	0·58	5·38
10 ……	93·1	0·84	0·70	1·28	0·71	0·88	8·19

IRON.

No. 22 : s.w.g. iron wire supplied by Ormiston Bros.

Density = 7·796 gm./c.c.

Constant of apparatus = 0·009308.

Initial load on short wire = 155 gm.

" " long " = 70 "

Heat Treatment. Wires heated by a current of 10·4 amps. and slowly cooled, *i. e.* to a temperature of 500° C. approximately.

Mean diameter = 0·712 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	V.	Mean Values.	Actual excess extension of shorter wires (10^{-3} cm.).
1 ……	1·68	—	—	—	—	—	—	—
2 ……	3·35	—	—	—	0·02	0·02	0·01	0·07
3 ……	5·03	0·02	0·02	0·03	0·05	0·05	0·03	0·32
4 ……	6·70	0·05	0·05	0·06	0·07	0·08	0·06	0·58

Heat Treatment (cont.).

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	V.	Mean Values.	Actual excess extension of shorter wires (10^{-3} cm.).
5	8·38	0·09	0·08	0·09	0·10	0·12	0·10	0·89
6	10·06	0·13	0·11	0·12	0·14	0·17	0·13	1·25
7	11·73	0·19	0·17	0·18	0·20	0·23	0·19	1·81
8	13·41	0·26	0·25	0·25	0·25	0·30	0·26	2·44
9	15·08	0·34	0·32	0·32	0·32	0·38	0·34	3·13
10	16·76	0·50	0·46	0·47	0·46	0·54	0·49	4·52
11	18·44	0·72	0·60	0·58	0·70	0·72	0·65	6·09
12	20·11	1·20	0·86	0·90	1·18	1·04	1·06	9·83

NICKEL.

No. 24: s.w.g. nickel wire supplied by Ormiston Bros.

Density = 8·834 gm./c.c.

Constant of apparatus = 0·009344.

Initial load on short wire = 96 gm.

" " long " = 48 "

Heat Treatment. Wires heated by a current of 10 amps. and slowly cooled, *i.e.* to a temperature of 500° C. approximately.

Mean diameter = 0·561 mm.

Load (kgm.).	Load on shorter wire. (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	V.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
0·5	1·35	—	—	—	—	—	—	—
1·0	2·69	0·01	0·02	0·02	0·02	0·02	0·02	0·17
1·5	4·04	0·10	0·10	0·09	0·09	0·09	0·09	0·88
1·75......	4·71	—	0·14	0·14	—	0·14	0·14	1·31
2·0	5·39	0·20	0·20	0·20	0·19	0·20	0·20	1·85
2·25......	6·06	0·32	0·29	0·28	0·28	0·30	0·29	2·75
2·5	6·73	0·43	0·41	0·39	0·39	0·41	0·41	3·79
2·75......	7·41	0·57	0·60	0·54	0·60	0·59	0·58	5·42
3·0	8·08	0·79	0·92	1·10	—	0·98	0·95	8·85

BRASS.

No. 24 : s.w.g. hard brass wire supplied by Ormiston Bros.

Density = 8·410 gm./c.c. Initial load on short wire = 156 gm.

Constant of apparatus = 0·009308. ,, ,, long ,, = 71 ,,

1. *Heat Treatment* A. None; wire used as supplied.

2. *Heat Treatment* B. Wires heated by a current of 8 amps. and slowly cooled, *i. e.* to a temperature of 300° C. approximately. Mean diameter = 0·560 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
1	2·70	—	—	—	—	—
2	5·40	0·05	0·04	0·04	0·04	0·40
3	8·11	0·15	0·11	0·11	0·12	1·15
4	10·81	0·30	0·25	0·23	0·26	2·42
5	13·51	0·50	0·42	0·39	0·44	4·07
6	16·21	0·70	0·60	0·59	0·63	5·86
7	18·91	0·91	0·82	0·80	0·84	7·85
8	21·62	1·16	1·10	1·08	1·11	10·36
9	24·32	1·46	1·44	1·42	1·44	13·42
10	27·02	1·83	1·80	1·86	1·83	17·04

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	IV.	Actual excess extension of shorter wire (10^{-3} cm.
0·5	1·35	—	—
1·0	2·70	0·20	1·86
1·5	4·05	0·50	4·65
2·0	5·40	0·90	8·38
2·5	6·76	1·25	11·64
3·0	8·11	1·65	15·36
3·5	9·46	2·20	20·48
4·0	10·81	2·90	27·00

Experiments I., II., and III. were performed on wires having heat treatment A, and experiment IV. after heat treatment B.

Copper : *Specimen* 1.— Electrolytic.

No. 24 : s.w.g. bright copper wire supplied by the London Electric Wire Company and Smith's.

Density = 8·887 gm./c.c. Initial load on short wire = 96 gm.

Constant of apparatus = 0·009344. " " long " = 48 "

1. *Heat Treatment* A. Wires heated by a current of 8 amps. and slowly cooled, *i.e.* to a temperature of 300° C. approximately. Mean diameter = 0·560 mm.

2. *Heat Treatment* B. Wires heated by a current of 11 amps. and slowly cooled, *i.e.* to a temperature of 570° C. approximately. Mean diameter = 0·559 mm.

Load (kgm.).	Load on shorter wire (kgm./sq. mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	V.	VI.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).	Scale Deflexions (cm.). VII.	VIII.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
0·0	0·00	—	—	—	—	—	—	—	—	—	—	—	—
0·5	1·35	0·25	0·14	0·13	0·20	0·16	0·19	0·16	1·50	0·10	0·10	0·10	0·93
1·0	2·71	0·66	0·35	0·32	0·44	0·38	0·40	0·43	4·02	0·38	0·40	0·39	3·64
1·5	4·06	1·24	0·62	0·58	0·75	0·64	0·73	0·76	7·10	15·2	15·8	15·5	154·83
2·0	5·41	1·98	0·95	1·20	1·67	1·05	1·20	1·34	12·52				

As the agreement between the values was not too good, the specimen of wire being particularly difficult to free from kinks, experiments were also performed on a second specimen.

Copper: *Specimen* 2.

No. 24: s.w.g. copper wire supplied by Ormiston Bros.

Density = 8·955 gm./c.c.

The constant of the apparatus and the initial loads were the same as for the first specimen.

Heat Treatment. Wires heated by a current of 10·5 amps. and slowly cooled, *i.e.* to a temperature of 500° C. approximately.

Mean diameter = 0·557 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
0·5	1·37	—	—	—	—	—	—
0·75	2·05	0·00	0·03	0·00	0·00	0·01	0·07
1·0	2·74	0·03	0·05	0·03	0·06	0·04	0·40
1·25	3·42	—	0·14	—	0·16	0·15	1·40
1·5	4·10	0·20	0·25	0·21	0·28	0·24	2·20
1·75	4·79	0·46	0·45	0·49	0·48	0·47	4·39
2·0	5·47	0·96	1·10	0·89	0·86	0·95	8·90
2·25	6·16	2·22	—	1·89	—	2·06	19·21

Aluminium.

No. 24: s.w.g. aluminium wire supplied by Ormiston Bros.

Density = 2·694 gm./c.c.

Constant of apparatus = 0·009344.

Initial load on short wire = 96 gm.

" " long " = 48 "

Heat Treatment. Wires heated by a current of 6·7 amps. and slowly cooled, *i. e.* to a temperature of 300° C. approximately.

Mean diameter = 0·564 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	V.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
0·5	1·34	—	—	—	—	—	—	—
1·0	2·67	—	0·14	0·14	0·14	0·13	0·14	1·31
1·5	4·00	0·56	0·52	0·54	0·53	0·52	0·53	4·95
2·0	5·34	1·08	1·16	1·18	1·20	1·14	1·15	10·74
2·5	6·67	2·68	2·71	2·32	2·47	2·60	2·56	23·92

ZINC.

No. 22 : s.w.g. zinc wire supplied by the London Electric Wire Company and Smith's Ltd.

Density = 7·181 gm./c.c.

Constant of apparatus = 0·009344.

Initial load on short wire = 96 gm.

" " long " = 48 "

Heat Treatment. Wires heated by a current of 9·6 amps. and slowly cooled, *i. e.* to a temperature of 300° C. approximately.

Mean diameter = 0·722 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	V.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
0·0	0·000	—	—	—	—	—	—	—
0·1	0·163	0·04	0·06	0·05	0·06	0·06	0·05	0·50
0·2	0·326	0·16	0·22	0·20	0·20	0·20	0·20	1·83
0·3	0·489	0·30	0·40	0·40	0·40	0·40	0·38	3·55
0.4	0·652	0·62	0·66	0·66	0·66	0·65	0·65	6·07
0·5	0·815	1·00	1·00	1·00	1·00	1·00	1·00	9·34
0·6	0·978	1·44	1·42	1·44	1·46	1·44	1·44	13·46
0·7	1·141	1·95	1·92	1·98	1·98	1·97	1·96	18·31
0·8	1·304	2·78	2·80	2·80	2·80	2·80	2·80	26·13
0·9	1·466	3·86	3·90	3·92	3·90	3·90	3·90	36·40
1·0	1·629	6·82	10·08	6·68	6·70	6·72	7·40	69·14

Tin.

No. 22: s.w.g. tin wire supplied by the London Electric Wire Company and Smith's Ltd.

Density = 7·292 gm./c.c.

Constant of apparatus = 0·009344.

Initial load on short wire = 53 gm.

" " long " = 26·5 "

Heat Treatment. Wires heated by a current of 6·2 amps. and slowly cooled, *i. e.* to a temperature of 200° C. approximately.

Mean diameter = 0·718 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	IV.	V.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
0·00	0·000	—	—	—	—	—	—	—
0·03	0·050	0·07	0·08	0·07	0·07	0·07	0·07	0·67
0·05	0·082	0·17	0·19	0·17	0·17	0·17	0·17	1·63
0·07	0·116	0.31	0·35	0·33	0·27	0·33	0·32	2·97
0·10	0·165	0·63	0·69	0·63	0·65	0·65	0·65	6·07
0·12	0·198	0·93	1·05	1·00	0·99	1·01	1·00	9·32
0·14	0·231	2·43	5·75	1·97	2·22	2·07	2·89	26·98

Lead.

No. 22: s.w.g. lead wire supplied by the London Electric Wire Company and Smith's Ltd.

Density = 11·344 gm./c.c.

Constant of apparatus = 0·009344.

Initial load on short wire = 53 gm.

" " long " = 26·5 "

Heat Treatment. Wires heated by a current of 5 amps. and slowly cooled, *i.e.* to a temperature of 200° C. approximately.

Mean diameter = 0·720 mm.

Load (kgm.).	Load on shorter wire (kgm./sq.mm.).	Scale Deflexions (cm.). I.	II.	III.	Mean Values.	Actual excess extension of shorter wire (10^{-3} cm.).
0·00	0·000	—	—	—	—	—
0·02	0·033	0·16	0·16	0·16	0·16	1·50
0·05	0·082	0·42	0·47	0·43	0·44	4·11
0·07	0·115	0·69	0·75	0·70	0·71	6·66
0·10	0·164	1·28	1·23	1·25	1·25	11·70
0·12	0·197	1·72	1·62	1·62	1·65	15·45
0·15	0·246	2·33	2·23	2·25	2·27	21·21
0·17	0·278	3·13	2·70	2·69	2·84	26·54
0·20	0·328	3·71	3·63	3·70	3·68	34·38
0·22	0·360	4·39	4·73	4·75	4·6 2	43·19

Discussion of Results.

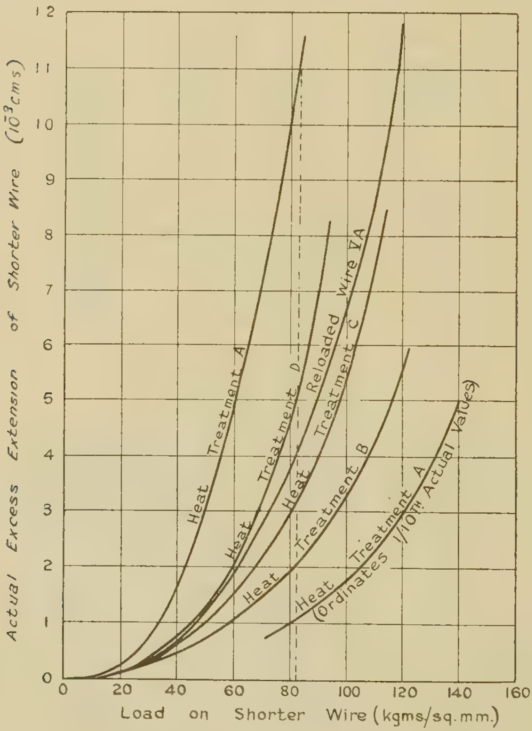
The results show that in no case is Hooke's law obeyed, excess extension of the shorter wire being observed for all loads (above that necessary to keep the wires taut), the amount of the deviation depending on the material and its treatment.

Ferrous metals.—The results for steel are shown graphically in fig. 3, the latter part of the curve for unheated steel (*i.e.* heat treatment A) being drawn with ordinates one-tenth their real size so that all the curves can be reasonably accommodated. The effect of annealing at 280° C. approximately (heat treatment B) is seen to be the reduction of the excess extension to one-fourth or one-fifth of its previous value for corresponding loads. Heating at higher temperatures, approximately 400° C. and 500° C. (heat treatments C and D), produces small increases in the excess extension for the same load, and, as will be shown later, the relation between the excess extension and load breaks down for smaller loads with increasing annealing temperature. The curve for experiment V. *a*, in which the wires were reloaded after they had been strained by a load equivalent to

165 kgm./sq.mm. on the shorter wire, shows a similar decrease in the value of the excess extension.

This is the general agreement with the change of properties of severely cold-worked steel upon subsequent annealing (*vide* Adam[(6)]). "There is distinct evidence that some of the effects of cold work are not permanent, even at ordinary temperature. Mere ageing undoubtedly influences the

Fig. 3.



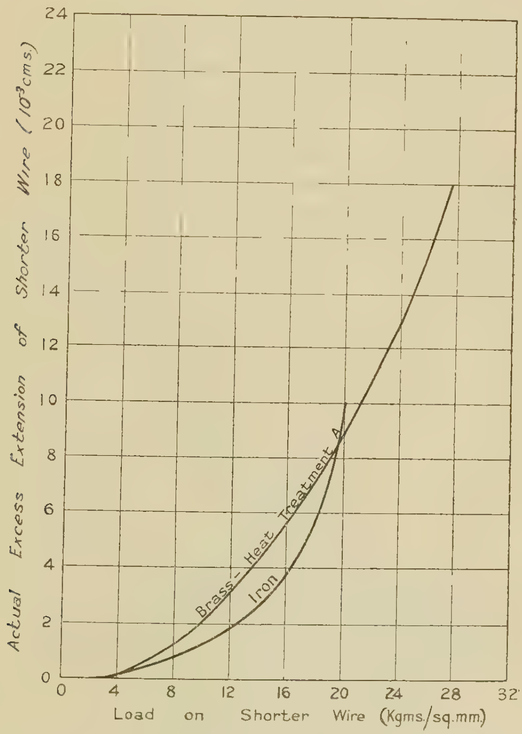
Excess extension-load curves for steel.

physical properties to some extent. The changes become more pronounced, however, as the temperature increases; thus, boiling in water produces distinctly measurable alterations in the elastic properties, and the limit of proportionality has been raised to over 80 per cent. of the ultimate strength in a cold-drawn wire heated to just under 400° C. There is little doubt that the modulus of elasticity is also affected by heat treatment after cold work. In short, up to 400° C.

there is a distinct stiffening or hardening effect, but above this point the cold-worked metal begins to soften again." If this limit of proportionality is considered to be that stress at which the curve for total elongation deviates 0·025 per cent. from a straight line (Government Specification), then this statement is in accordance with the present observations.

The bright drawn steel wire (sorbitic pearlite) being a severely cold-worked material has a micro-structure of small

Fig. 4.



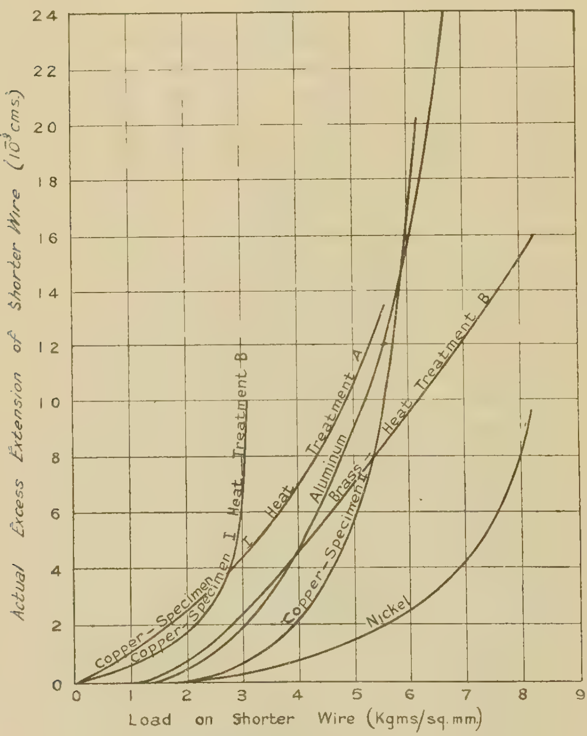
Excess extension-load curves for iron and brass.

elongated crystals, and even the highest annealing temperature used would leave the sorbite and pearlite areas distorted.

The results for iron are shown in fig. 4; it will be noted that the excess extension is very much greater than that for steel for similar loads. However, in this case the annealing temperature (approximately 500° C.) is sufficient to cause recrystallization, *i. e.* large size of grain.

Non-ferrous metals.—The behaviour of unheated brass (heat treatment A) is shown in fig. 4; the values of the excess extension are numerically comparable with those of iron. The material had been subjected to considerable cold work, being hard and extremely brittle. On annealing at 300° C. approximately (heat treatment B) the brass became soft and the excess extension very much greater, being

Fig. 5.



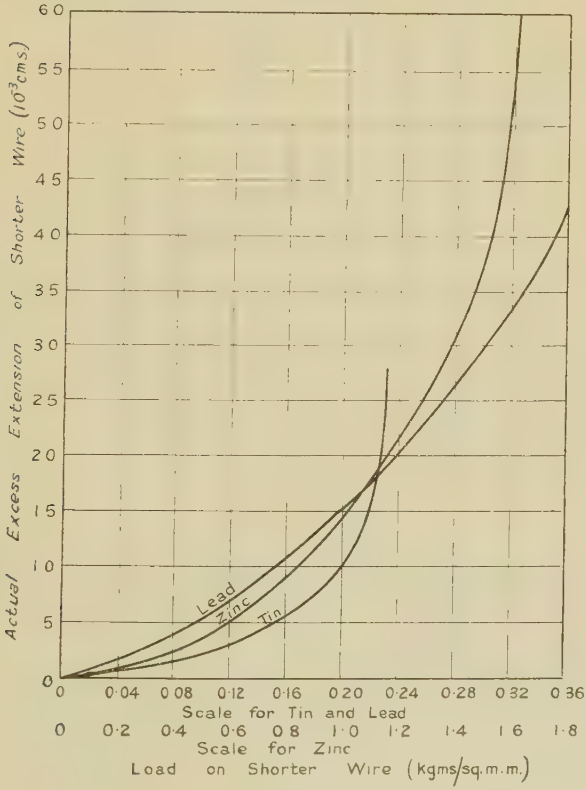
Excess extension-load curves for nickel, copper, aluminium, and brass.

similar in behaviour to copper. Fig. 5 shows the results for copper, aluminium, nickel, and the heated brass. Copper, specimen 1, was a bright drawn wire annealed at two different temperatures, 300° C. and 570° C. approximately (heat treatments A and B), the deviations from Hooke's law being much greater for the wires heated to the higher temperature. Finally, fig. 6 shows the curves for zinc, tin,

and lead, the abscissæ scale for zinc being one-fifth of that for tin and lead.

It is well known that if a metal subjected to internal stress is heated, crystal growth occurs, and, other things being equal, the higher the temperature the more complete the recrystallization and ultimate crystal size.

Fig. 6.



Excess extension-load curves for lead, tin, and zinc.

Hence, the brass and copper experiments indicate that for annealed metals the amount of deviation from Hooke's law is a function of the crystal size, suggesting that the deviation is intimately connected with the metal crystal.

Some corroboration of this view is given by the experiments of Thompson quoted by Adam (*loc. cit.* p. 168),

showing the variation with crystal size of the "true elastic limit"—a quantity, presumably, as defined above.

Dead Mild Steel.

Carbon 0·05 per cent.

Treatment.	No. of crystals per cm.	True elastic limit (tns. per sq.in.).
Annealed at 900° C.	276	9·61
Normalized.........................	343	12·0
Oil quenched at 900° C. and tempered at 650° C.	690	14·0

Open-hearth Steel.

Composition: carbon 0·26, silicon 0·05, manganese 0·57, sulphur 0·043, phosphorus 0·054 per cent.

Treatment.	No. of crystals per cm.	True elastic limit (tns. per sq.in.).
Drastically over annealed......	168	less than 4·0
Normalized at 850° C.	416	15·0
Oil quenched at 850° C. and tempered at 350° C.	500	17·5

Summary.

(1) It has been found that all metals investigated show deviations from Hooke's law over the whole experimental range.

(2) The amount of the deviation differs widely for different materials, and even for the same material varies greatly for different heat treatments and cold-working.

(3) For annealed metals the deviation increases with increase of crystal size.

Acknowledgment.

In conclusion, this work was performed at East London College and I have much pleasure in thanking Professor Lees for the facilities he has placed at my disposal and for his interest and encouragement during its progress. I also wish to express my thanks for a grant from H.M. Department of Scientific and Industrial Research.

References.

(1) Tyte, Phil. Mag. x. p. 1043 (1930).
(2) Thompson, *Ann. der Phys.* xliv. p. 555 (1891).
(3) Grüneisen, *Verh. phys. Ges.* p. 469 (1906); *Phys. Zeitschr.* vii. p. 901 (1906); *Ann. der Phys.* xxii. p. 81 (1907); xxv. p. 825 (1908).
(4) Schülze, *Ann. der Phys.* xxxi. p. 1 (1910).
(5) Kyrillov, *Jurn. Russk. Fisik-Chimicesk Obscestva*, xxxix. No. 3, p. 64 (1907).
(6) Adam, 'Wire Drawing and the Cold Working of Steel,' p. 145 (1925). Witherby.

V. *On the Interaction of Radiation and the Electron.*

By R. D. Kleeman, *B.A.*, *D.Sc.**

§ 1. *Proposed Properties of the Electron.*

THE nature of the connexion between the electron and radiation is one of the outstanding problems of modern physics. Some of the experiments on the subject have led to the supposition that radiant energy does not continuously fill space, but is localized, the radiation being supposed to proceed in the form of darts. But this is almost impossible to reconcile with interference. According to other experiments the electron under certain conditions seems to behave like a wave-train, and under other conditions like a particle. In this paper we will show that certain of these apparently conflicting results, retaining the electro-magnetic theory in its orthodox form, may be reconciled if we assume that the electron and proton possess the following properties :—

(1) An electron or proton may possess internal energy apart from kinetic energy.

(2) The internal energy, or a part of it, may be converted directly into radiation, or apart from acceleration, on account of a disturbance such as a collision etc., and the process may take place according to the equation

$$\Delta u = h\nu, \quad \ldots \ldots \ldots \quad (1)$$

where Δu denotes the change in internal energy, ν the frequency of the radiation emitted, and h Planck's constant.

* Communicated by the Author.

(3) Radiation falling upon an electron or proton is absorbed in indefinite amounts, or not necessarily in quanta, which becomes internal energy. Temperature radiation is absorbed when the charge is in motion.

(4) The absorption of temperature radiation has a retarding effect on the motion of the charge.

(5) The force acting on a charge placed in an electric field depends on its internal energy, and when this increases the force in a general way decreases.

(6) Electrical potential energy of a charge may be converted into internal energy, and *vice versâ*.

(7) The laws of conservation of energy and momentum hold when an electron and radiation interact.

The last-named property may be taken to follow from our general knowledge of the kinetic properties of matter. Evidence that the properties (1), (3), (4), (5), (6), and the first part of (2) should hold were obtained by the writer * from considerations of the equation of a perfect gas obtained thermodynamically, and from the distribution of the velocities of the particles in a gas. Further evidence was obtained † in connexion with the application of a magnetic field to an electron gas, the axiomatic assumption ‡ that we cannot have an infinite amount of energy per c.c., the application of radiation to an electron gas §, and more evidence of a similar nature which will be published shortly.

The results obtained have been used by the writer to interpret the Bohr atom ||. It was shown that an electron describing a Bohr orbit in an atom gradually slows down till in a stationary position on the orbit. During the process it absorbs energy from the surrounding radiation which is stored up as internal energy. It will now not be under the action of a force as far as the other charges of the atom are concerned, except when displaced from its stationary position. If an electron passes from its position to one corresponding to a smaller internal energy content, which lies on another Bohr orbit, the difference in the internal energies is emitted *directly* into

* Phil. Mag. vii. p. 493 (1929); *Z. anorg. allgem. Chem.* cxcvi. p. 284 (1931); *Z. Elektrochem.* xxxvii. pp. 78, 371 (1931).

† 'Nature,' cxxiv. p. 728 (1929) 'Science,' lxx. p. 478 (1929).

‡ 'Science,' lxxi. p. 340 (1930).

§ 'Science,' lxii. p. 225 (1930).

|| Phil. Mag. vii. p. 493 (1929).

space. The frequency of the radiation emitted (following Bohr) obeys equation (1) *.

The difficulty of the nature of the process of radiation and that of the non-radiating Bohr orbit now disappear from the Bohr atom. Further, since the atom has now changed from a dynamic to a static nature, it no longer conflicts with the Lewis-Langmuir theory of the atom.

It appears also from the static Bohr atom that the force F between two electrical charges e_1 and e_2 separated by the distance r should be written

$$F=\frac{e_1 e_2}{r^2}\cdot\phi(e_1, e_2, u_1, u_2, r), \quad . \quad . \quad . \quad . \quad (2)$$

where u_1 and u_2 denote the internal energies of the charges and ϕ is a function of the quantities e_1, e_2, u_1, u_2, and r. It appears, furthermore, from the behaviour of the charges in a Bohr atom that F may be zero for certain values of u_1, u_2, and r †.

We shall now explain by means of the foregoing results three of the main experiments which have proved a difficulty in the orthodox electromagnetic theory of radiation.

§ 2. *The Velocity of the Electrons emitted from Materials subjected to Radiation.*

If radiation of a frequency ν is allowed to fall upon a material, it has been found by experiment that the velocity v of the electrons ejected is given by the equation

$$h\nu=\tfrac{1}{2}mv^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where m denotes the mass of an electron. Furthermore, the number ejected per sq. cm. varies inversely as the square of the distance of the material from the source of the rays. Experiments on this relation have been carried out mainly with ultra-violet light and X-rays.

To explain it let us consider first of all an electron having a store of internal energy equal to $h\nu$. It will evidently be a system possessing a resonance frequency ν, and therefore if radiation of that frequency is allowed to fall upon it

* For other properties of the electron, see *Z. f. Phys.* lxiii. p. 859 (1930); lxix. p. 286 (1931).

† The connexion of an atom of this nature with various physical and chemical properties has been pointed out by the writer in papers in *Z. anorg. u. allgem. Chem.* cxci. p. 201 (1930); cxciii. p. 106 (1930); cxcv. p. 164 (1931); cxcix. p. 225 (1931).

the internal energy is likely to be emitted into space as radiation of frequency ν.

Next consider two electrons separated initially by an infinite distance, one of which is brought up to a finite distance of the other. The external work performed may be written $h\nu$. Now suppose that this work, or the resultant increase in potential energy of the displaced electron, becomes internal energy. Since the total change in potential energy becomes internal energy, the work done upon the electron in transporting it again to an infinite distance would be zero. It will appear now that if potential energy and electronic internal energy may be converted into each other according to the electronic property (6), the property (5) will also hold, and *vice versâ*.

Next, suppose that the electron in the position to which it was brought (whose store of internal energy is accordingly $h\nu$) is subjected to radiation of a frequency ν, in which case it will be set radiating by resonance. This process would increase the field of the electron, and thus give rise to an increase in potential energy equal to the energy radiated, according to what has gone before. But this is impossible, or the two processes could not take place simultaneously; and hence no energy will be radiated, but converted into potential energy. The energy will in turn be converted into kinetic energy of the electron according to equation (3) if allowed to move reely to infinity under the action of the recovered field.

Finally, consider a Bohr atom in which the electrons have assumed stationary positions. On account of the motion of the atom as a whole each electron will continually absorb radiant energy which increases the internal energy content. The stationary position of each electron may in consequence be displaced; but a reverse displacement may also (periodically) occur, induced by radiation or some other disturbance, in which case some of the internal energy is radiated into space according to equation (1). But it may also happen that if the total available internal energy of the electron is $h\nu$, and this is equal to the potential energy it would possess on recovering its field, the internal energy may be converted into potential energy, and this in turn into kinetic energy according to equation (3), on radiation of frequency ν falling upon the electron. Thus the existence of equation (3) is explained. The underlying principles may be stated briefly in the form

that if an electron possesses an amount of internal energy equal to $h\nu$ which may become potential energy of repulsion, the process of conversion may be induced by radiation of frequency ν falling upon it.

It is evident that other possibilities in the emission of secondary electronic radiation suggest themselves. They will be treated at length in a separate paper.

The elemental amount of radiation absorbed by an electron in an atom to set it radiating at the same frequency is likely to be the same for each electron. The number of electrons liberated per c.c. in a material by a radiation of given frequency is therefore simply proportional to the energy passing through the material. The inverse square law of ionization then immediately follows.

§ 3. *The Compton Effect.*

When X-rays are allowed to fall upon a material they are scattered more or less in all directions, and undergo besides a change in wave-length according to A. H. Compton. To explain this effect he supposes * that an X-ray may be looked upon as an entity of energy $h\nu$, which may collide with a free electron as if it consisted of a particle, and that during the collision the principles of conservation of energy and momentum hold, and that the mass of the radiation and that of the electron are determined by the Principle of Relativity. These considerations give the equation

$$\Delta\lambda = \cdot 0484 \sin^2 \tfrac{1}{2}\theta, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $\Delta\lambda$ denotes the increase in wave-length in passing from a direction of observation in line with the original rays to one making a direction θ with it. This equation has been found to agree well with the facts. It may, however, also be explained, as will now be pointed out, by means of the properties of the electron stated in § 1.

A free electron initially at rest may gradually absorb energy from the impinging beam of X-rays, not necessarily in quanta, till the energy absorbed is equal to $h\nu$, which is represented by the sum of the increase in internal energy of the electron and the increase in its kinetic energy $\frac{1}{2}mv^2$, where v denotes its final velocity, which is in the same direction as the radiation. It is now a system whose natural frequency is ν, and hence, on being further

* Phys. Rev. xxi. pp. 207, 483 (1923).

subjected to radiation of this frequency the acquired internal energy is radiated into space. This may take place in a small solid angle whose direction makes an angle θ with the original direction of the rays. This process can now easily be shown to give the same equation as (4).

It may be urged as an objection that if the orthodox electromagnetic theory holds the time of absorption of a quantum of energy $h\nu$ would be so large that it could not fail to affect the course of the experiments. But actually such an absorption is not necessary. We have seen that a free electron, on account of its motion, continually absorbs radiant energy, which is stored up as internal energy and which is periodically emitted again as radiation on being subjected to a disturbance. It will therefore happen that when X-rays of frequency ν fall upon a material some of the free electrons will have a velocity v in the direction the rays are moving *, and some will possess besides a sum of available internal energy and kinetic energy equal to or greater than $h\nu$. These would be induced by the X-rays to emit, as before, the amount of internal energy $h\nu-\frac{1}{2}mv_1^2$ in the form of radiation in a small solid angle in a direction making an angle θ with the X-rays, where v_1 denotes the final velocity of the electron.

There are evidently many other possibilities that may be predicted in this connexion †.

§ 4. *The Reflexion of Electrons from Crystals.*

De Broglie has suggested that an electron which has the velocity v represents a wave-train of frequency ν given by the equation

$$\frac{h\nu}{c}=mv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where c denotes the velocity of light. Experiments carried out by Davisson and Germer ‡ have within certain limits proved the truth of this equation. They found that cathode rays of velocity v reflected from a crystal behave

* Mostly for a short time only, since the kinetic energy of the electron at the end of its free path may be largely converted into potential energy and this into internal energy.

† Thus, for example, a stationary electron having the internal energy content $2h\nu$, might be induced to radiate towards the source of the exciting radiation a beam whose energy is $h\nu$, and a beam in the opposite direction according to equation (4).

‡ Nat. Ac. of Science, xiv. p. 317 (1928).

like X-rays of frequency ν given by the above equation. Slight deviations, however, occur from the above equation under certain conditions it should be mentioned, and sometimes the reflected beam is entirely missing. But the result is notwithstanding a striking one in the light of equation (5). It is, however, difficult to see how physically an electron may consist of a wave-train and still retain its identity during the process of reflexion. For this requires that the train of waves be broken up into at least two parts which are reflected from two different layers of atoms. The result may, however, more logically be explained on the basis of the results stated in § 1.

When the electron penetrates into the crystal it may eject a beam of rays of frequency ν and energy $h\nu = cmv$ in the same direction as that in which it is moving. This beam may be reflected from the layers of atoms of the crystal and then be again absorbed by the now stationary electron, which then moves off with the original velocity v. Equation (5) would then be satisfied. But evidently the physical process cannot be as simple as that.

Let us therefore next suppose that the angle of incidence of the ejected radiation is less than that of the moving electron. This will in consequence not be reduced to rest, and it may now meet the reflected pencils of radiation where they join together to form the original beam. On absorbing the beam the electron would regain its original velocity and proceed in a direction corresponding approximately to that as if the electron consisted of a wave-train of frequency ν. The meeting of the reflected pencils and the electron may be ensured, since the process of reflexion of the radiation by the atoms may consist of an absorption and emission with an interval between. This would help to explain besides many anomalies in the reflexion of electrons.

But actually the electron will not emit radiation during the process of reflexion; for as soon as this begins to happen the corresponding energy will simultaneously, altogether or in part, be converted into potential energy, similar to what happens in the process described in § 2. But this is impossible. Internal energy is therefore converted into potential energy instead of radiant energy, and *vice versâ*, or the law of force of the moving electron is temporarily changed, and this takes place in such a manner that the electron is reflected as if it consisted of a wave-

train of approximate frequency ν. It is easy to see now that deviations from the law expressed by equation (5) may occur, and that these may be quite large under certain conditions.

In this paper the interaction of radiation and the electron is treated from a somewhat different point of view than that put forward by de Broglie and Schrödinger, who suppose that the electron is a packet of radiation. But the two views are by no means antagonistic to each other; as a matter of fact, if an electron consists of a packet of radiation it is all the more likely to absorb and emit radiant energy in the way described. Possibly in the end a combination of the various principles involved to supplement each other might prove the desirable thing. Since the general properties of the electron deduced from thermodynamics do not involve any new theorems or assumptions, they should form the cornerstone in these developments.

127 Woodland Av.
Schenectady, N.Y., U.S.A.

VI. *Characteristic Curves of the Aluminium Rectifying Cell.* *By* L. L. Barnes, *M.Sc., A.K.C.**

THE aluminium cell is a serviceable rectifier for such uses as charging storage batteries from A.C. supply, and some of its characteristics are outlined as an indication of the manner in which the optimum conditions for its working can be determined.

The cell under test consists of aluminium and lead electrodes in a solution of sodium borate.

Characteristic Curve of Current and Potential.

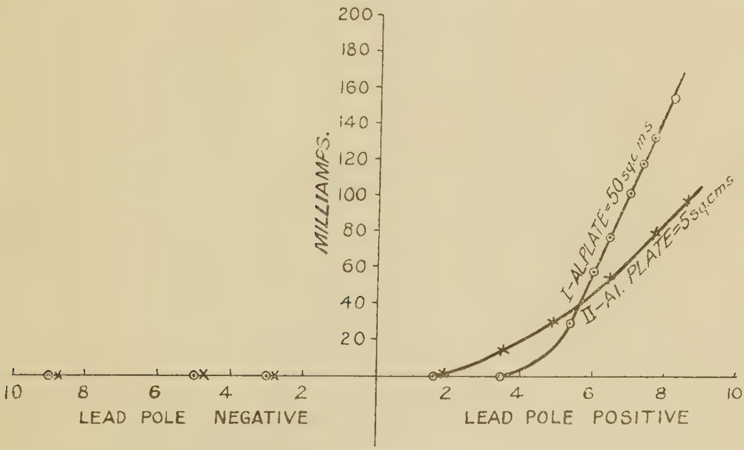
The rectifying property is most clearly shown by the characteristic curves in fig. 1 (Tables I. and II.). Curve I. is obtained using an aluminium pole of submerged area equal to 50 sq. cm., and Curve II. with area of aluminium equal to 5 sq. cm.

The area of lead electrode does not affect the characteristic except in so far as it governs the purely ohmic resistance

* Communicated by the Author.

between the lead and the electrolyte. Fig. 2 shows the circuit by which the curves are obtained. The switch S is moved in one position to allow alternating current to pass through the cell (*via* the resistance R). It is then moved to the other position, and direct current from the battery B is measured by the milliameter A and the potential of the battery by the voltmeter V. The A.C. is then again applied in order to maintain the rectifying film on the aluminium) before taking the next measurements for a different value of potential from the battery. The current noted is that recorded by A instantaneously on switching over to D.C., as the direct current does not remain constant, but slowly rises.

Fig. 1.



Characteristic curve of the rectifier.

TABLE I.—Aluminium electrode = 50 sq. cm.

Lead pole positive.		Lead pole negative.	
Volts.	Milliamps.	Volts.	Milliamps.
1·8	0	3	0·0
3·5	0	5	0·0
5·4	30	9	0·0
6·0	58		
6·4	77		
7·0	102		
7·4	120		
7·8	137		
8·3	155		

The resistance of the cell to current flowing in the direction aluminium to lead is extremely high, and even at an applied potential of 100 volts is as much as 150,000 ohms.

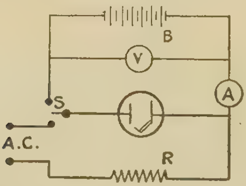
Effect of Area of Aluminium Plate.

The effect of variations in the area of the aluminium electrode and in temperature are examined by measuring

Table II.—Aluminium electrode = 5 sq. cm. Characteristic—D.C. Current and Potential.

Lead pole positive.		Lead pole negative.	
Volts.	Milliamps.	Volts.	Milliamps.
2·0	2	3	0·0
3·5	15	5	0·0
4·8	30	9	0·0
6·5	55		
7·7	78		
8·7	100		

Fig. 2.

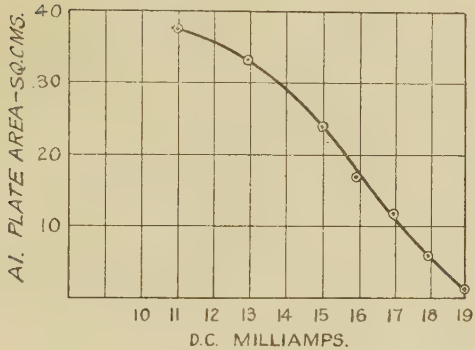


Circuit for obtaining characteristic curve of the rectifier.

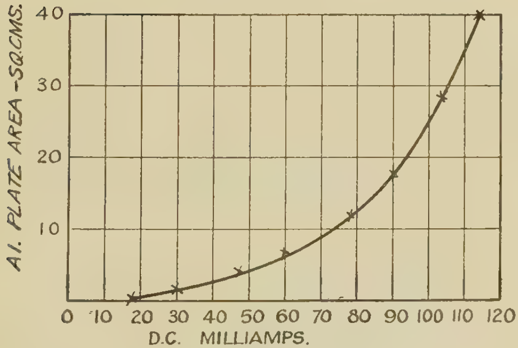
the rectified (half-wave) current for a fixed applied A.C. potential.

The curves in fig. 3 (Tables III. and IV.) are obtained by measuring the rectified current while varying the area of aluminium plate by drawing the electrode out of the electrolyte. Curve I. is taken with an impressed A.C. voltage of 5 volts, and curve II. with 8 volts. The fact that the curves slope in opposite directions shows that for smaller rectified currents the area of aluminium plate should be kept small, while for larger outputs the area should be greater. This is confirmed by reference to the characteristic curves in fig. 1, which show that up to currents of 40 milliamp. the 5 sq. cm. plate gives the more efficient rectification, while above 40 milliamp. the 50 sq. cm. plate is the more efficient.

Fig. 3.



Curve I.—5 volts A.C.



Curve II.—8 volts A.C.

Effect of variation of aluminium plate area.

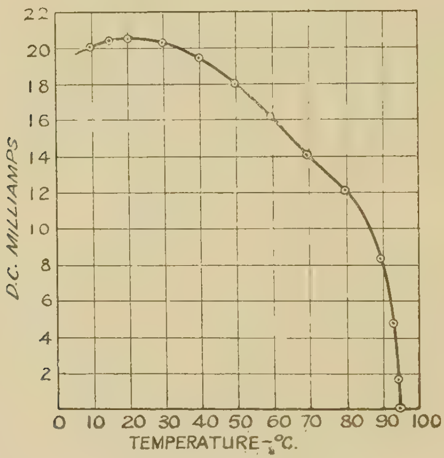
TABLE III.—5 volts A.C. TABLE IV.—8 volts. A.C.
Rectified Current and Aluminium Plate Area.

Milliamps.	Sq. cm.	Milliamps.	Sq. cm.
11	37·5	18	0·9
13	33	30	2·1
15	24	48	4·2
16	17	60	6·9
17	12	78	12·0
18	6	90	19·8
19	1·5	103	28·5
		114	40·2

Effect of Temperature.

Raising the temperature of the cell reduces its power of rectification. The curve in fig. 4 (Table V.) is obtained by

Fig. 4.



Effect of temperature on D.C. output.

TABLE V.—Rectified Current and Temperature.

° C.	Milliamps.
10	20·0
15	20·4
20	20·6
30	20·3
40	19·5
50	18·0
60	16·2
70	14·2
80	12·2
90	8·4
93	4·8
94	1·6
95	0·0

keeping the impressed voltage and the area of aluminium constant and heating the cell externally. [The area of aluminium plate in this case is 50 sq. cm. and the applied A.C. voltage 5.]

It is noticeable that a slight heating-up of the cell is not undesirable; experiment shows that this is due to the reduction in resistance of the electrolyte with rise of temperature more than counteracting the falling-off of rectification. In the case investigated, after 25° C. the rectification begins to decrease rapidly, and ceases completely at 95° C.

7 Redwing Lane, Norton-on-Tees,
Co. Durham.

VII. *The Effect of Mechanical Working on the State of a Solid Surface.* *By* R. F. HANSTOCK, *B.Sc.*, *A.Inst.P.**

COLD-working is known to change the physical properties of an annealed metal. Kelvin showed that a thermo-couple could be made by using two wires of the same metal, one wire being annealed by heating and the other cold-worked by drawing or hammering. This was later verified by Maclean [1], the E.M.F. generated being of the order 0·1 millivolts per centigrade degree, the current flowing generally from the annealed to the strained metal at the hot junction. Since then many other physical properties have been shown to depend on the degree of deformation of the substance. Beilby [2] repeated the thermoelectric experiment, and showed that the E.M.F. disappeared after the wires had been heated to a temperature which was about one-third of the absolute temperature of the melting-point of the metal. In the same way the elasticity of a metal depends on its previous history as regards cold-working and permanent strain. Among other properties changed by cold-working are:—

(*a*) The electrical conductivity [3]. This is usually lower in the strained than in the annealed metal.

(*b*) The magnetic susceptibility [4]. The diamagnetic susceptibility of metals belonging to the cubic system decreases on cold-working. The susceptibility of copper is changed from diamagnetic to paramagnetic by cold-working.

* Communicated by the Author.

(*c*) The Laue X-ray diagram shows distortion for strained metals [5].

(*d*) Transparent substances become doubly refracting when strained [6].

It is evident from the above that deformation leads to considerable change in the physical properties of a substance.

Similarly it is known that a change can be produced in the surface layers of a metal by parallel but less drastic methods, *e. g.*, polishing and rubbing. In order to detect such a change it is necessary to study properties of the material which are quite different from those most influenced when the material is deformed in bulk. This is natural, since the substance as a whole is not appreciably influenced by polishing its surface. The effect of surface treatment will be rendered apparent mainly by those phenomena which are intimately connected with surface conditions.

The investigation of the effects of mechanically working a surface is considerably more difficult than for the corresponding examination of substances deformed in bulk, owing to the activity of surfaces in adsorbing gases and in forming layers of oxides and impurities. On account of this the amount of positive evidence of the formation of a layer of "strained" substance by working the surface is limited. There is, however, a certain amount of indirect evidence of such a change. In the following, reference is made to published researches which indicate that a change is produced in the nature of a surface when it is mechanically worked. It is difficult to be certain whether this change is to be attributed to a deformation of the crystal lattice comparable with that observed in treatment *en masse* or simply to disturbances and change of adsorbed films.

Microscopic examination was made by Beilby [2] of polished metallic surfaces after they had been treated with an etching liquid. He arrived at the conclusion that polishing produced a "vitreous" layer on the surface of the metal. Etching showed that the "vitreous" was more soluble than the crystalline material. A change from the "vitreous" to the crystalline state was observed on heating the surface to about one-third of the absolute temperature of the melting-point of the substance. This vitreous or amorphous layer, produced by polishing, was concluded to be of the same nature as the amorphous substance observed to form between the crystalline grains when a metal was strained in bulk, and it was suggested that the formation

of such material was due to partial liquefaction under pressure of the crystalline grains of the material. After liquefaction the solidification took place so rapidly that the metal appeared in a non-crystalline or vitreous phase. The vitreous was stronger than the crystalline material, and acted as a cement between the crystalline grains, thus giving the material as a whole a greater strength when strained than when annealed.

The theory of polishing suggested by Beilby was that a layer of this "vitreous" material was formed on the surface by liquefaction of the crystalline grains under pressure of the polishing tool. From etching observation it was shown that the vitreous layer was formed to depths of from 50 to 500 $\mu\mu$. Disturbing effects were of course produced by the polishing tool at greater depths.

This work on polishing provides the most direct evidence of the production, by mechanical working, of surface layers having different physical properties from the substance in bulk form.

Assuming the validity of the deduction that the surface layer produced by polishing is of the same nature as the substance formed between the crystalline grains of the metal when cold-worked, and that the changes in the various physical phenomena previously noted are due to the formation of such amorphous material, it is to be expected that the layer produced by polishing will affect considerably those phenomena which are most sensitive to change in surface conditions. Among those most likely to be affected by the formation of such a layer the following immediately suggest themselves: friction between solids, contact potential, frictional electricity, photoelectricity, and surface reflexion of radiation.

Friction.

Since friction probably arises in part from cohesion of the two surfaces in contact, and will thus depend on the structure of the crystal lattice at the points of contact, any deformation of the lattice by mechanical working may be expected to alter the coefficient of friction. R. B. Dow [(7)] measured the coefficient of friction for rods sliding *in vacuo*, and showed that it depended largely on the condition of the surfaces. He found that the coefficient of friction μ increased with the number of times the rods slid over one another, and attributed the change in μ to the formation of a vitreous layer. The experiment was carried out in air at a reduced pressure of 1·0 mm. and without heating. In later work by Shaw and

Leavey [8] the coefficient of friction was measured for rods heated up to 350° in air at a pressure of ·01 mm. They found a difference in μ for strained and annealed surfaces, μ being greater for the annealed than for the strained metals. This is exactly opposite to the effect found by Dow, but is probably a more reliable result, since the experiment is performed in better conditions.

Contact Potential.

The dependence of contact potential on surface conditions has been investigated *in vacuo* by Ende [9], who found that potentials up to 0·5 volts could be produced between similar metals. The metal surfaces were treated with acid, emery paper, worked with a harder metal, sawn, drawn, and filed. No systematic variation was noted, and the effects were attributed to differences in adsorbed gas layers. It seems possible, however, that the contact potential may in part be due to actual differences in the state of strain of the surfaces. A further point to be noted is that the adsorbed gas layer may be different for annealed and strained surfaces.

Reflecting Power.

Margenau [10] has investigated the dependence of the ultra-violet reflexion of silver on the state of the surface. The minimum of reflexion for silver occurs in the near U.V. (3160 Å.), and was examined after treating the surface in different ways, *e. g.*, after vigorous and moderate polishing, etching, and preparing by depositing the silver electrolytically. It was shown that for highly polished plates the minimum was very low, and situated at 3160 Å. For an unstrained surface it was shifted to lower wave-lengths by 20 Å, and had a considerably higher value than for the polished plate.

Triboelectricity.

The present research is concerned mainly with the effects of mechanical working as revealed by triboelectricity and photoelectricity, and these will be considered in detail.

In 1917 Shaw [11] showed that the quantity and sometimes the sign of the charges developed when two substances were rubbed together depended to a great extent on the previous treatment of the surfaces. Two states of the surface were distinguished, termed "normal" and "abnormal," and it was shown that the position of a substance in the tribo-electric series was determined by the state of the surface.

The "abnormal" could be produced from the "normal" state by heating the substance to some temperature, usually about 250° C. The experiments were performed in air and were of a purely qualitative nature. An important point was discovered when it was shown [12], contrary to general belief, that charges could be generated by rubbing together two rods of the same material. When one rod was rubbed along another similar rod it was found that they became charged negatively and positively respectively. Continued rubbing, however, was found to influence the surfaces in such a way as to reverse the signs of the charges on the rods. This effect was permanent unless the rods were heated to some definite temperature, when they were restored to their original state. It was suggested that the strained state, *i. e.*, that produced by much rubbing, corresponded to the "normal" state in the previous work and that this changed on annealing to the state previously called "abnormal."

It was from this point that the present research was commenced, an endeavour being made to study quantitatively the changes arising in the frictional charge generated when two similar bodies were rubbed together. It is convenient at this point to give a definition of the term "mechanical working" as used in this paper. By "mechanical working" is meant the bringing of two solid surfaces into intimate contact. This may occur in the following ways:—

(*a*) Simple contact with pressure.

(*b*) Contact by rubbing.

(*c*) Contact by impact, normal or glancing.

Normal impact is probably only a more drastic form of simple contact with pressure. So far it is found that the method (*b*) is most fruitful in results.

Triboelectric Experiments.

An earlier paper [13] contains an account of the method employed in finding the relation between the triboelectric charge and the amount by which a surface has been mechanically worked. For the sake of clarity a brief *résumé* is given here. The simplest case to consider is that in which two similar insulating solids are rubbed together. If this could be done so that both solids were mechanically worked to the same degree, *e. g.*, two perfectly plane disks of the same area rubbed together with a rotary motion about an axis through their centres, there would be no reason to

expect a separation of charge. It is impossible to produce this degree of perfection in practice, and some separation of charge always takes place. Where there is pronounced differentiation in the amount of mechanical working of the two surfaces considerable charges arise. The experimental arrangement used in the previous paper is that of two cylinders in contact with their axes at right angles; one cylinder (called the rubber) is made to slide over the other (called the rubbed), so that one spot on the first cylinder is always in contact, and slides for a definite distance along a generator of the second cylinder. The apparatus allows a known pressure to be applied between the two cylinders. The charge arising on the rubbed cylinder on separating the surfaces is indicated by a sensitive gold-leaf electroscope.

The experimental method is to anneal the two cylinders by heating to a suitable temperature, and then to take readings of the charge separated after each rub of one cylinder over the other. The cylinders are discharged after each reading. Conditions of temperature and humidity are regulated, the humidity being kept as low as possible, when it is found to have no appreciable effect on the charge separated.

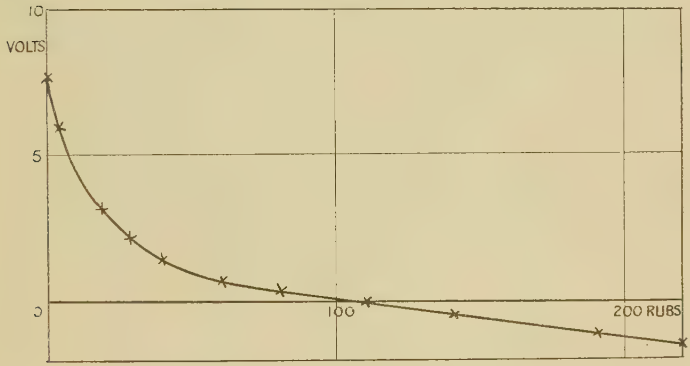
The materials used in the experiment are ebonite and celluloid as the insulators, and various metals. It is unfortunate that we are restricted to the use of such materials as ebonite and celluloid, but this is due to the difficulties arising in these experiments. The materials used must not be too hard, or very small charges are produced, presumably owing to the small amount of deformation; also they must be of such a nature as to resist crumbling and flaking on rubbing. These restrictions limit the number of insulators which it is possible to use to these two. Most of the results are obtained using ebonite as the insulator, this being better on the whole than celluloid. There are a number of metals that may be used, but most of the work is done with copper and zinc.

Figs. 1, 2, and 3 show typical curves for frictional charges arising between ebonite-ebonite, zinc-ebonite, and copper-ebonite as the amount of mechanical working increases (the first material of the pair represents the rubbing body in each case, *i. e.*, the one having the most mechanical working).

The use of ebonite presents one complication. It is shown [13] that the temperature at which there is a relaxation of strain for ebonite (of the quality used) is low. Complete relaxation takes place in 30 minutes at 100° C. When one piece of ebonite is made to rub over another a certain

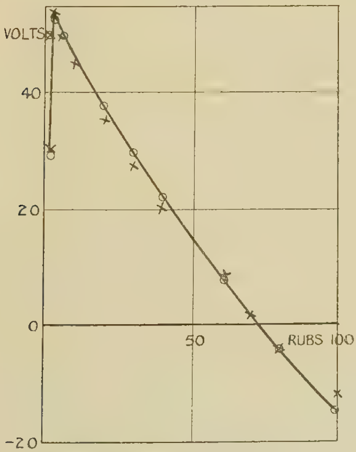
amount of heat is generated. Since ebonite is a poor heat conductor it is probable that the temperature attained by the small area of the rubber is such as to cause a certain amount of relaxation after the rubbing. This, however, does not

Fig. 1.



Ebonite-ebonite at 75° C.

Fig. 2.

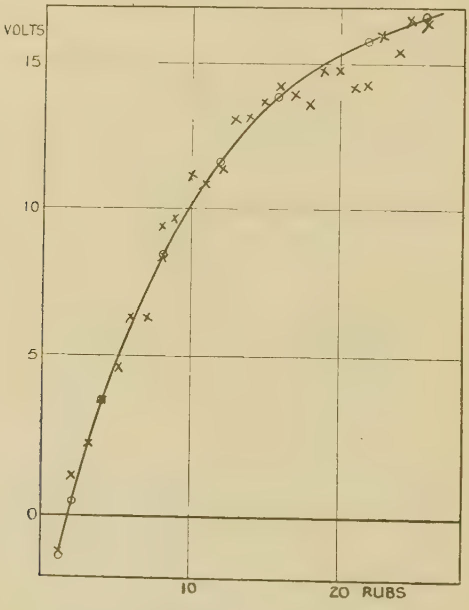


Zn-Ebonite at 60° C.

apply to so great an extent to the rubbed ebonite, since for this the area mechanically worked is much larger, and for any point on this area there is contact with the rubber for only a very short time. This relaxation effect is probably negligible for metals when they are rubbed on ebonite, since

these are good conductors of heat and also have much higher temperatures of relaxation. With due regard to this it is possible to give a qualitative explanation of the curve obtained for ebonite-ebonite (fig. 1). Since the surfaces are originally in the same condition (annealed by boiling together in water) the starting-point of the curve must be the origin (0, 0), there being no reason why electric separation should take place between similar bodies in simple contact. During the first rub a positive charge is

Fig. 3.



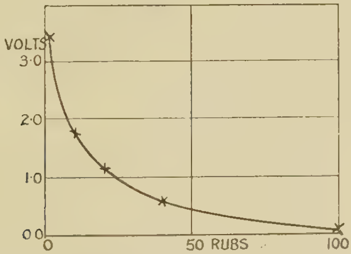
Cu-Ebonite at 90° C.

obtained and the rubber becomes more mechanically worked than the rubbed ebonite. Thus the charge produced by mechanical working, whatever its exact nature, is such as to make ebonite charge negatively against other ebonite mechanically worked to a lesser degree. In most cases the charge produced by the second is less than that produced by the first rub, indicating that the surfaces are becoming more like one another, *i. e.*, the rubber is changing less rapidly than the rubbed ebonite. Since the area of the rubber is much less than that of the rubbed, it is probable that the

change in the surface of the former reaches its limit during the first rub. Prolonged rubbing does not, as might be expected, lead to the two surfaces reaching the same condition. If this were so the curve would approach the zero line of charge asymptotically. What is actually observed after prolonged rubbing is the separation of a constant small negative charge on the rubbed body. This indicates a state of greater change for the rubbed ebonite than for the rubber, and is in agreement with the idea that the rubber suffers a certain amount of relaxation between the rubs, due to heating during contact with the rubbed body.

In a previous paper [13] it is demonstrated that the amount of relaxation of strain for a substance like ebonite depends not only on the temperatures of the ebonite but also on the time elapsing after straining. Thus, if a series of rubs are given the amount of relaxation of the rubber depends on the

Fig. 4.



Ebonite-ebonite at 59° C. No relaxation.

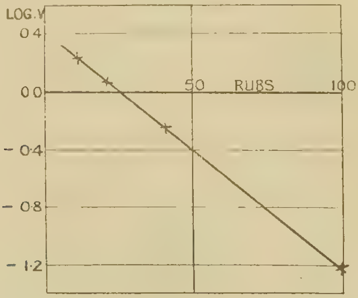
intervals between the rubs. If the rubs follow one another fairly rapidly it is found that the amount of relaxation of the rubber becomes negligibly small. It is impossible, however, during a series of rubs to observe the charge and then discharge between each rub, with time intervals so small that no relaxation takes place. This difficulty may be overcome by the following procedure. The first rub is made, and the charge observed in the usual way. The next point on the curve is obtained for, say, ten rubs by starting again, with fresh places on the ebonites and giving nine rubs rapidly discharging while rubbing, and then observing the charge for the tenth rub in the usual way. Other points on the curve are obtained in a similar manner. Fig. 4 shows a curve obtained in this way, and it is seen that it now approaches the axis of zero charge asymptotically, showing that the relaxation effect has been reduced to a negligible amount.

Except for the point given by the first rub this curve may be represented by an equation of the form

$$Q = Ae^{-\lambda m},$$

where Q is the charge for any given rub m. Fig. 5 shows values of log Q plotted against m, a good straight line being obtained if the first point is neglected. It has already been stated that for ebonite-ebonite the charge obtained for the second rub is less than for the first, indicating that the rubber is approximately strained to the limit (*i. e.*, the state in which further rubbing produces no change in the tribo-electric properties) during the first rub, so that, providing no relaxation takes place, the subsequent part of the curve represents the effect of straining the rubbed body only. Thus we may say that the change in the condition of an

Fig. 5.



Ebonite-ebonite at 59° C. No relaxation.

ebonite surface on rubbing is such as to make the charging dependent on the rubbing according to the relation $Q = Ae^{-\lambda m}$. In general the curve obtained when two pieces of ebonite are rubbed together, assuming relaxation to be eliminated, is one represented by the difference of the two exponential curves appropriate to the two surfaces. In practice this is simplified by the fact that the expression for the rubber usually reaches its limit during the first rub.

The curves obtained for copper-ebonite and zinc-ebonite suggest that the same relation may be true for these metals. It is found that the curves for zinc-ebonite may be represented by an equation of the form

$$Q + A = B(e^{-\lambda_1 m} - e^{-\lambda_2 m}),$$

where A, B, λ_1, and λ_2 are constants and λ_2 is much greater than λ_1. Except for the appearance of the constant A this

is of the same form as the equation for ebonite-ebonite, the term $Be^{-\lambda_2 m}$ representing the change of surface condition of the rubber. This cannot be neglected for zinc, since it is less easily strained than the ebonite. The change in the surface of the zinc does not reach its limit in the first rub, as happens when ebonite is used. The curve shown in fig. 2 for zinc-ebonite at 60° C. is found to be represented by

$$V + 101{\cdot}5 = 157{\cdot}3\,(10^{-0{\cdot}0026m} - 10^{-0{\cdot}781m}),$$

V being in volts.

In the figure the circles show points calculated on the equation, the crosses being experimental observations.

From the results obtained for zinc-ebonite at different temperatures it is found that the position of the peak of the curve depends on the temperature. For 22° C. it appears at the fifth rub, for 60° C. at the third rub, for 70° C. at the second rub, and for 90° C. no peak is attained on the curve, showing that the zinc becomes fully strained during the first rub. If it is assumed [14] that plastic flow in metals begins at a temperature approximately one-third of the absolute melting-point this is explicable. The melting-point of zinc is 418° C., which gives −43° C. as the temperature at which plastic flow commences. Hence a variation of temperature from 22° C. to 90° C. should produce an appreciable effect on the rate at which the zinc is strained by rubbing.

The curve for Cu-ebonite is seen to be, in the main, a mirror image of that for zinc-ebonite, and may be represented by an equation of the form

$$Q - A = B\,(e^{-\lambda_1 m} - e^{-\lambda_2 m}),$$

only in this case λ_1 is much greater than λ_2. Indeed, for the curve for copper-ebonite at 90° C. the peak has disappeared and the term $e^{-\lambda_1 m}$ has become negligible after the first rub. The peak is quite apparent for copper-ebonite at 60° C. The curve for copper-ebonite at 90° C. may be represented by the equation

$$V - 18{\cdot}2 = 22{\cdot}2\,.\,10^{-0{\cdot}044m}.$$

In this case it appears to be the ebonite (*i. e.*, the rubbed body) which is strained to the limit first. This is borne out by the fact that the strain is more rapidly developed at 90° C. than at 60° C., since the peak appearing in the curve for 60° C. is not present in that for 90° C. The copper, although having a much smaller area in contact (since it is the rubber) changes more slowly than the ebonite. The temperature at which plastic flow commences for copper is about 179° C.,

which is well above the temperature of the experiment. It is to be noted that iron-ebonite gives a curve of the same form as copper-ebonite.

Consideration of the general equation given above shows that A is proportional to the charge which should arise on contact of the two materials without rubbing. It is impossible to produce charges of this order by simple contact, probably because the contact ordinarily obtained is not sufficiently intimate. Experiments have been performed with iron and ebonite in which normal impact took place between two surfaces, and the results compared with the ordinary rubbing experiments for iron-ebonite. Allowing for the differences in areas involved, it is found that the charge developed by normal impact is of the same order as that produced by the first rub in the ordinary experiment.

It will be seen that for ebonite-ebonite A is zero (or very nearly so), as would be expected. For zinc-ebonite A is negative, while for copper-ebonite it is positive. The charges, however, are those appearing on the ebonite, so that, considering the charges appearing on the metals, A is positive for zinc and negative for copper. According to this, if copper and zinc are placed in contact the copper should assume a negative potential with respect to the zinc. This is in accordance with previous observations. Also it appears probable that the potential difference between annealed copper and annealed zinc will be the same as between copper and zinc when strained to their limits, since the constant B disappears for both $m=0$ and $m=\infty$. This may not be strictly true, since it cannot be determined with sufficient accuracy whether the coefficients of the two exponential terms in the full equation are exactly equal.

Thus the general conclusions to be drawn from these experiments are that:

(1) A mechanically worked surface charges negatively when rubbed against a surface of the same material which is mechanically worked to a lesser degree.

(2) The charge produced when a surface is rubbed on another surface which is in such a condition as to be unchanged by the rubbing is an exponential function of the number of rubs.

(3) Between similar bodies there is no separation of charge on intimate contact without rubbing.

(4) No separation of charge arises on rubbing between

similar bodies when they are mechanically worked to the limit (relaxation effects being eliminated).

(5) Between unlike bodies a charge should be separated on intimate contact without rubbing, the charges being such as to agree with the contact-potential between the two bodies.

(6) The charge separated between two unlike annealed bodies is probably the same as the charge separated between the same bodies when mechanically worked to the limit.

Photoelectric Experiments.

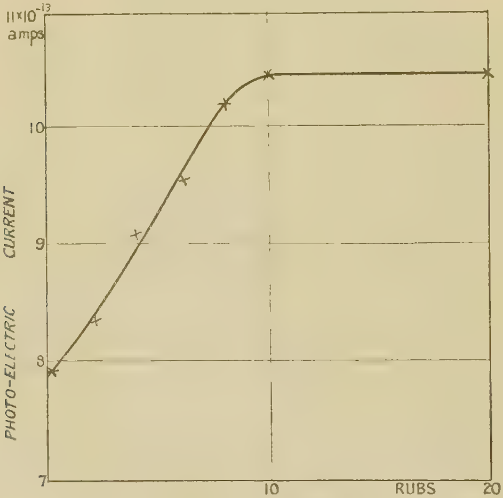
X-ray examination indicates a difference between the structures of annealed and cold-worked metals when these are treated in bulk, but fails to reveal any change in the surface layer of a polished metal. This is probably because X-rays penetrate to a depth far greater than that influenced by the working of the surface, so that the X-ray diffraction pattern is that of the bulk of the metal rather than of the surface layer. From experiments on thin metallic films short wave-length U.V. radiation is known to penetrate to and release electrons at a depth (30 $\mu\mu$) comparable to that influenced by the mechanical working of the surface. It is therefore reasonable to suppose that the liberation of photo-electrons from the surface of a metal will depend on the amount of mechanical working of the surface.

Apparatus for testing this is described in detail in a previous paper (15). Briefly, the apparatus consists of a photoelectric cell in which the illuminated surface is a thin flexible strip of metal. By a mechanical device the strip can be polished with a steel cylinder sliding over its surface. The cell is surrounded by a heating coil, so that the metal may be annealed *in vacuo.*

The following is a summary of the results obtained. The specimens tested are ribbons of copper, silver, gold, and platinum, and are mechanically worked *in vacuo* by sliding a steel or nickel cylinder over them. The curves obtained relating the photoelectric current to the amount of mechanical working are similar for all the metals tested and are of the form shown in fig. 6. The curve is not of simple exponential form, but is approximately linear during the increase of the photoelectric current, the surface becoming more sensitive when mechanically worked. Annealing the metals at 300° C. reduces the photoelectric current to its

original value. The annealing process begins at approximately the same temperature as that at which change in the triboelectric properties is observed. The ratio of the photoelectric current i_0 for the annealed metal to the current i_m for the worked surface shows no regular dependence on the pressure of the gas, and is of the order of 1·4 for Ag and 1·2 for Cu when polished with steel. The effect appears to be independent of the character of the radiation. It cannot be stated definitely that the effect is due to the formation of a mechanically worked layer on the surface, yet it is probable that the change, at least in part, is due to such a layer.

Fig. 6.



Cu–Fe. Press.$=$0·40 mm.

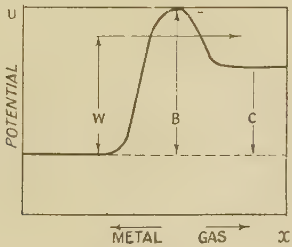
Nordheim [16] has shown that the probability of an electron passing through the surface of a metal depends on the transmission coefficient of the surface. For a metal situated *in vacuo* there is a layer at the surface in which the potential increases very rapidly in such a way that work must be done to take an electron through the surface to the outside. This corresponds to the quantity P in the Einstein equation for the photoelectric emission,

$$\tfrac{1}{2}mv^2 = h\nu - \mathrm{P}.$$

If W is the component normal to the surface of the energy of electrons released by the incident U.V. radiation, the electrons are totally reflected if $\mathrm{W} < \mathrm{P}$ and totally

transmitted for $W >> P$. If, however, the potential relation at the surface is not of the simple form considered above, but is modified owing to the presence of a surface layer differing in properties from the metal as a whole, the transmission conditions are altered. If, for instance, the layer is of material less electropositive than the metal underneath, the potential change will be of the form shown in fig. 7, and it has been shown that there is a definite probability of an electron being emitted when its energy W lies between B and C (see fig. 7). Thus, for a definite frequency of the incident light the electron emission would be increased by such a layer. If therefore the effect of mechanical working is to produce a layer of metal which is less electropositive than the metal underneath, an increase in the emission is to be expected on rubbing. It is of course

Fig. 7.



possible that the same effect would be produced by the formation of a layer of oxide on the surface. It is difficult to say whether the progressive formation of such a layer would produce the observed relation between the photoelectric current and the amount of mechanical working.

If mechanical working does result in the formation of a less electropositive layer on the surface it should be possible to interpret the results of the triboelectric experiments in terms of this change. From these experiments it is shown that the body which is the more strained tends to charge negatively against one which is less strained. This seems to require the formation by mechanical working of a layer which is more electropositive than the annealed substance. Thus, interpreting the increase of electron emission in the photoelectric experiments as due to the formation of a less electropositive layer on the surfaces leads to direct conflict with the triboelectric experiments, although it must be remembered that the passage of electric charge across the

interface takes place in entirely different conditions in the two experiments. In the photoelectric experiments a change is observed in the electron emission from metal into vacuum, while in the triboelectric experiments flow of charge takes place between two surfaces in relative motion, the surfaces being either two insulators or an insulator and a metal. The charging during triboelectric experiments may not be due solely to electron flow across the interface, but also to the passage of charged particles split off the surface by the violent rubbing action.

On the other hand, it is possible that the change observed in the photoelectric emission is not due mainly to a change in the work of emission of electrons. Margenau [(10)] has shown for silver that the reflecting power depends on the state of plastic deformation of the surface layer. He finds that for wave-lengths below about 3200° A. (*i. e.*, for light of the wave-length used in the photoelectric experiments) the reflecting power is greater for an annealed than for a highly polished surface. Thus for wave-length 2000 Å. the reflecting power of an etched surface is 0·4, while for a highly polished surface it is 0·3. It is suggested that a similar effect may be present in the photoelectric experiments described in this paper. Assuming a constant intensity of incident radiation, a greater fraction of this would be reflected for an annealed than for a polished surface. Thus there would be a greater proportion of light producing photo-electrons for the polished than for the annealed surface.

Relaxation Temperatures.

Experiments have been made to determine the relaxation temperatures of some metals, as indicated by changes in the triboelectric and photoelectric properties. According to Desch [(14)] plastic flow in metals begins at a temperature which is approximately one-third of the absolute melting-point of the metal. If this is true, relaxation from strain would be expected to begin at about these temperatures. The following table shows the melting-points of various metals, the corresponding temperatures of commencement of plastic flow, and the relaxation temperatures as determined from triboelectric and photoelectric experiments.

In the triboelectric experiments the relaxation temperatures were determined as follows:—The metal surface was strained to the limit by burnishing. The condition of the surface was then tested by rubbing on ebonite after the metal had been heated to some definite temperature and allowed to cool.

This was done several times, through a range of temperatures. From these readings the temperature at which the tribo-electric charge first showed change was estimated.

In the photoelectric experiments a similar method was employed, the photoelectric current from the polished metal surface being observed after the specimen had been heated to increasingly higher temperatures.

Experiment.	Metal.	Melting-point.	$\frac{A^\circ}{3}-273$.	Observed relaxation temperature.
Triboelectric	Cu......	1084° C.	179° C.	170° C.
	Fe......	1530	328	200
	Cd......	321	− 75	< room temperature.
Photoelectric	Ag......	962	139	120° C.
	Au......	1063	172	< 250
	Cu......	1084	179	< 250
	Pt	1710	388	300

Thus there is fairly good agreement between the temperatures at which change is found in triboelectric and photoelectric phenomena and the temperatures at which plastic flow commences. The observed temperatures appear to be rather lower than those calculated. It is suggested that this may be due to the presence of a comparatively large mass of the annealed metal underlying the deformed surface layer. This underlying annealed matter may help the annealing process to start in the surface layer at a lower temperature than it would if the metal were deformed throughout.

For cadmium it was found that a small amount of relaxation took place at ordinary room temperatures. This is in agreement with other observations on similar plastic metals. Rosenhain (17) has shown from ball-hardness tests on severely strained lead that slight relaxation takes place at ordinary temperatues after a few hours. Tin and zinc are also stated to undergo softening at ordinary temperatures.

Conclusions.

The experiments described in this paper and the results of other investigators in similar subjects seem to show fairly conclusively that the effect of mechanically working a surface is to produce a layer differing in physical properties from the underlying annealed material. Beilby thought of this layer as being produced by actual liquefaction of the

annealed crystalline material under pressure. This conclusion was arrived at from observation with the microscope. The phenomena observed seemed to indicate a liquefaction of the metal when polished, yet it must be remembered that the observations were really of macroscopic dimension compared with the lattice units of the metal. It is probable that the apparent liquefaction observed for Beilby may be explained simply as due to twisting and gliding of the lattice planes under pressure of the polishing tool. From experiments on simple metal crystals it has been shown that the properties such as photoelectricity and thermoelectricity depend on the particular face of the crystal dealt with. It seems probable that the changes observed in the physical properties discussed may, at least for the metals, be explained as due to a change in orientation of the crystal faces at the surface produced by mechanical working. A certain amount of orientation may exist on the surface of a substance owing to the intense electric field existing at the interface.

The present research throws some light on the difficult subject of triboelectricity in that it indicates that the separation of charge which takes place when two substances are rubbed together depends primarily on the contact potential between the bodies (this assumes the existence of a Volta effect for insulators). The potential difference on which the separation of charge depends may be altered by mechanically working the surface—in fact it is in general impossible to produce frictional charges without at the same time mechanically working the surface.

The experiments on relaxation temperatures afford evidence that the results obtained in the triboelectric and photoelectric experiments are related to the experiments of Beilby and others, who find changes produced by the mechanical working of surfaces.

This research was made possible by facilities afforded by the Physics Department of University College, Nottingham.

Summary.

A brief account is given of published work on this and allied subjects. The experimental research is mainly an investigation of the effect of mechanically working a surface as indicated by changes in the triboelectric and photoelectric properties of the surfaces. It is concluded that in these subjects, as well as in those investigated by others, the condition of the surface as influenced by mechanical working

plays an important part in determining the magnitude of the effect studied. The curves obtained in the triboelectric experiments seem to indicate the existence of a definite Volta contact effect between the interfaces insulator/insulator and metal/insulator. This contact potential is influenced by the changes produced by mechanically working the surfaces. The changes produced by mechanical working may be made to disappear by heating to some definite relaxation temperature. In the triboelectric and photoelectric experiments the relaxation temperatures are approximately the same as those at which plastic flow is supposed to commence in metals.

References.

(1) Maclean, Proc. Roy. Soc. (Feb. 1899).
(2) Beilby, 'Aggregation and Flow of Solids.'
(3) Michels, *Ann. der Physik.* lxxxv. p. 6.
(4) Honda and Shimizu, 'Nature,' cxxvi. p. 990 (1930).
(5) Joffé, 'The Physics of Crystals.'
(6) Coker, Cantor Lectures.
(7) Dow, Phys. Rev. xxxiii. (Feb. 1929).
(8) Shaw and Leavey, Phil. Mag. x. (Nov. 1930).
(9) Ende, *Phys. Zeit.* (Aug. 1929).
(10) Margenau, Phys. Rev. xxxiii. (June 1929).
(11) Shaw, Proc. Roy. Soc. A, xciv. p. 16.
(12) Shaw, Proc. Phys. Soc. xxxix. p. 447.
(13) Shaw and Hanstock, Proc. Roy. Soc. A, cxxviii. p. 474.
(14) Desch, Trans. Faraday Soc. xxiv. p. 57.
(15) Hanstock, Phil. Mag. x. p. 937 (1930).
(16) Nordheim, *Phys. Zeit.* xxx. (1929).
(17) Rosenhain, 'Introduction to Physical Metallurgy,' p. 269.

VIII. *A Note on the Sound generated by a Rotating Airscrew.* *By* E. T. PARIS, *D.Sc., F.Inst.P.**

§ 1. *Introduction.*

AN experiment is described which was performed with the object of obtaining evidence concerning the distribution of sound round a rotating airscrew for comparison with the theoretical conclusions of E. J. Lynam and H. A. Webb † and M. D. Hart ‡. The experiment consisted of the measurement, in various directions in a horizontal

* Communicated by the Author.

† "The Emission of Sound by Airscrews," Aero. Res. Comm., R. & M., No. 624 (1919).

‡ "The Aeroplane as a Source of Sound," Aero. Res. Comm., R. & M., No. 1310 (1930).

plane, of the amplitude of the fundamental tone of the "sound of rotation" of an airscrew on a stationary aeroplane.

It is convenient to begin with a statement concerning earlier work on airscrew sound in so far as it relates to the sound of rotation, and, in particular, to the distribution round the airscrew.

§ 2. *Earlier Investigations into the Sound of Rotation of an Airscrew.*

It is well known that one of the principal sounds generated by a rotating airscrew is an harmonic series of tones having a fundamental frequency equal to the product of the number of blades with the speed of rotation *. This sound is called the "sound of rotation," to distinguish it from other sounds generated by an airscrew. With the aid of a hot-wire microphone A. Fage † investigated the relative strengths of the harmonics of the sound of rotation generated by four-bladed model airscrews, and found that in general the amplitude decreased as the order of the harmonic was increased.

Lynam and Webb, in the course of a theoretical investigation into the effect of rotational speed on the amplitude of the sound generated, obtained results which indicated certain features of interest in connexion with the acoustical field round a rotating airscrew. They made use of two alternative hypotheses concerning the representation of a rotating airscrew as a source of sound.

Their first hypothesis (*loc. cit.* p. 5) was that the airscrew could be represented as rings of sources and sinks, the strengths of which varied periodically with a frequency equal to the fundamental frequency of the sound of rotation; the sources were supposed to be in front of the airscrew and the sinks behind it. The relative phases of the sources or sinks in each ring were determined by the motion of the airscrew in such a way that the phase corresponding to any selected position of the blade lagged behind that corresponding to an earlier position by an amount equal to the

* Messrs. Lynam and Webb and M. D. Hart, *loc. cit. supra*; R. McK. Wood, "Note on some Experiments on the Sound emitted by Airscrews," Aero. Res. Comm., R. & M., No. 694, p. 18 (1920); E. Waetzmann, "Die Entstehung und die Art des Flugzeugschalles," *Z. f. techn. Phys.* no. 6, p. 167 (1921); J. Obata, "The Analysis of the Sounds emitted by Aircraft," Rep. Aero. Res. Inst., Tokyo Imp. Univ., no. 59 (March 1930).

† "An Experimental Study of the Vibration in the Blades and Shaft of an Airscrew," Proc. Roy. Soc. A, cvii. pp. 456–458 (1925).

time taken by the blade to move from the earlier to the later position. These sources and sinks constituted in effect a system of doublets with their axes parallel to the axis of rotation of the airscrew. Lynam and Webb found that, on this hypothesis, the amplitude of the fundamental tone of the sound of rotation at a distant point (*i.e.*, at a distance great compared with the dimensions of the airscrew) due to a single ring of sources and the corresponding ring of sinks was proportional to

$$J_m\left(\frac{m\omega r}{a}\sin\beta\right).\sin\left(\frac{m\omega b}{a}\cos\beta\right), \quad . \quad . \quad . \quad (1)$$

where J_m = the Bessel function of order m;

m = number of airscrew blades;

ω = rotational speed of airscrew in radians per second;

r = radius of ring of sources or sinks;

a = velocity of sound;

β = angle between the line joining the point of observation to the centre of the airscrew and the axis of the airscrew;

and b = distance of rings of sources and sinks from the plane of rotation.

For any harmonic other than the fundamental, for example, the pth harmonic, m would be replaced by pm.

To obtain an expression for the amplitude of the sound in a selected direction due to the emission from the whole airscrew it would be necessary to assume some radial source distribution and to integrate expressions of the type (1) along the radius. To simplify the problem, however, Lynam and Webb assumed the sources and sinks to be concentrated "at one radius only, viz., $\frac{3}{4}$ of the bladelength, approximating roughly to the centre of pressure of the blade." They also assumed that $b=\frac{1}{2}r$. On these assumptions, and with L representing the blade-length, (1) becomes

$$J_m\left(\frac{3m\omega L}{4a}\sin\beta\right).\sin\left(\frac{3m\omega L}{8a}\cos\beta\right). \quad . \quad . \quad (2)$$

Lynam and Webb's second hypothesis (*loc. cit.* pp. 9 & 10) was that "while the sources are a little way in front of the airscrew, the sinks are a long way behind the airscrew, or for mathematical purposes at infinity." The problem then reduces to finding the effect at a distant point of a

single ring of simple sources having the phase-relationship referred to above, and the result, in the same notation as before, is

$$J_m\left(\frac{m\omega r}{a}\sin\beta\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If, as before, the sources are supposed to be concentrated into a single ring of radius 3L/4, the amplitude of the fundamental tone at a distant point should be proportional to

$$J_m\left(\frac{3m\omega L}{4a}\sin\beta\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The hypothesis that all the sinks were at infinity was regarded as an extreme case, and Lynam and Webb suggested that the "two hypotheses might be combined, one ring of sinks being taken a little way behind the airscrew and another ring of sinks at infinity, the relative intensities of the two sinks remaining a matter for further consideration."

M. D. Hart * has written a very full discussion of aeroplane sound, including a theoretical treatment of the sound from the engine exhaust openings and of that generated by the rotation of the airscrew. So far as the distribution of sound of rotation round an airscrew is concerned he arrived at an expression, identical with (3) above, for the amplitude at a distant point due to the disturbances from an annulus of radius r swept out by an element of the blade surface. The amplitude due to the motion of the whole blade would be given by an integral which cannot at present be evaluated. It appears, however, that the resultant sound-distribution would be symmetrical about the plane of rotation †.

§ 3. *The Sound Recording Apparatus.*

It was desired to concentrate attention on the fundamental frequency (about 27∼) of the sound of rotation of a certain airscrew, and for this purpose a tuned hot-wire microphone was used.

The resonator of the hot-wire microphone had a volume of 3345 c.c. The neck was cylindrical, 22 mm. long and 6 mm. in diameter. The damping factor (h) and the conductance of the neck (c) were 15·3 sec.$^{-1}$ and 0 0989 cm. respectively at 30∼. The experimental determination of these quantities has already been described ‡.

* *Loc. cit.*

† *Vide* the expression at the foot of p. 16 (Hart, *loc. cit.*).

‡ Proc. Phys. Soc. xliii. pp. 74–77 (1931).

The grid of the microphone was connected in one arm of a "battery bridge" circuit, with a compensating grid as a balancer.

Two galvanometers were used in parallel; one was of the Moll pattern, with a period of about $\frac{1}{4}$ second, and the other an Einthoven string galvanometer. The former served to record the average drop in resistance of the microphone grid, while the latter showed the alternating resistance-changes and produced a record from which the frequency of the sound affecting the microphone could be determined.

The deflexion of the Moll galvanometer was photographed continuously on a slow-speed recorder carrying a strip of sensitized paper moving at about $\frac{1}{8}$ inch per second. The shadow of the Einthoven galvanometer string was thrown on to the slit of a high-speed recording camera of the type employed with the Low-Hilger "audiometer." The shutter of this camera could be operated electromagnetically by the closing of a switch which, when records were being taken, was worked by an observer in the cockpit of the aeroplane.

The calibration of the microphone for the comparison of sound-amplitudes was effected by the stationary wave-method, whereby every observed change in average resistance δR (or the corresponding deflexion) could be interpreted as a relative pressure amplitude $\sin ky$, where $k = 2\pi/\text{wave-length}$ and y is the distance which the microphone must be displaced from a loop-position in a plane stationary wave in order to suffer the resistance change δR *.

The purpose of the frequency record was to make possible a correction for any variation in frequency of the sound which might occur between one observation and another. Since the microphone is a tuned instrument, any such variation would affect the magnitude of the steady resistance-change and might lead to false values for the amplitude. The correction was applied as follows:—

The observations yield a relative amplitude, $\sin ky$, and a frequency, n. Now if n_0 is the resonance-frequency of the microphone and q the amplitude in the neck of the resonator, then †

$$q = Q \cos \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where Q is the amplitude which would occur if the sound

* *Cf.* Proc. Phys. Soc. xxxix. pp. 274–275 (1927); xliii. pp. 75–76 (1931).

† Proc. Phys. Soc. xliii. p. 74 (1931).

were exactly in tune with the microphone (*i. e.*, if $n=n_0$), and α is defined by

$$\left.\begin{aligned}\alpha&=\tan^{-1}(\Delta/2h),\\ \Delta&=2\pi n_0(n_0/n-n/n_0).\end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (6)$$

Thus, if h, n, and n_0 are known, the relative amplitude ($\sin ky'$) that would have been recorded by the microphone had it been exactly in tune can be calculated from the formula

$$\sin ky'=\sin ky \,.\, \sec\alpha. \quad . \; . \; . \; . \; . \; . \quad (7)$$

The resonance-frequency (n_0) is dependent on the air-temperature, as shown by Rayleigh's formula,

$$n_0=\frac{a}{2\pi}\sqrt{\frac{c}{V}}, \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

a being the velocity of sound in air and V the volume of the resonator. The procedure adopted was to calculate n_0 from (8) by means of the known values of c and V on each occasion in which observations were made, the value of a appropriate to the observed air-temperature being used.

§ 4. *The Experimental Determination of the Distribution of Sound round the Airscrew.*

The aeroplane used in the experiment was a single-engined bomber with Rolls Royce "Condor" engine (700 h.p.), fitted with a two-bladed airscrew of diameter 4500 mm. (14·8 ft.) and pitch (pressure-face) 3200 mm. The gearing of engine to airscrew was 0·477. The engine was run at 1750 rev. per min. *, so that the calculated frequency of the fundamental tone of the sound of rotation was

$$(1750\times 0{\cdot}477\times 2)/60=27{\cdot}8\sim.$$

The aeroplane was on level open ground at a distance of 760 ft. from the microphone, and records were taken with the long axis of the fuselage making different angles with the direction to the microphone. For this purpose the tail of the machine was moved round between successive observations by steps of 15°. The boss of the airscrew was above the same point on the ground during each observation.

Since the plane of rotation of the airscrew when the aeroplane was resting on level ground was inclined at an

* This is below the normal cruising speed of 1850–1900 rev. per min., but was the highest which could be attained under the circumstances of the experiment.

angle of about 10° to the vertical, the angle β between the direction of the microphone (as seen from the airscrew boss) and the axis of rotation had to be worked out for each position. No observations were made in this experiment at points on the axis of rotation; the smallest value of β was 10°.

The microphone was supported on a frame at a height of 15 ft. This was to minimize possible errors arising from the upward refraction of sound-rays due to the temperature lapse-rate near the ground.

The procedure during the making of an observation was as follows:—The aeroplane being in position, with its engine running, an observer, with a switch communicating with the recording apparatus, took his place in the cockpit in a position whence he could see the engine revolution indicator. The pilot then accelerated the engine until the prearranged rev. per min. (1750) was reached, and maintained this speed as steadily as possible for a short period. During this time the observer closed the switch at what he considered a favourable moment, and a record was made.

The results of one complete set of observations are given in the following table:—

Distribution of Sound Amplitude (Airscrew Fundamental).

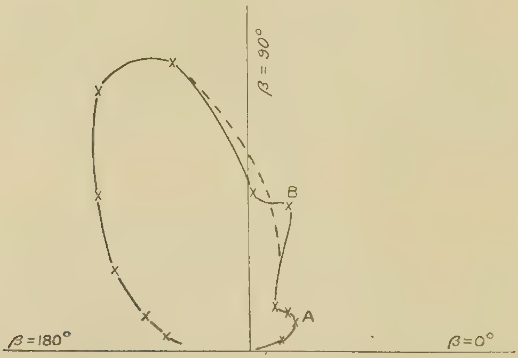
Record no.	Angle from axis of rotation (β).	Relative amplitude (sin ky).	Frequency (ω), cycle/sec.	Frequency correction factor (sec α).	Corrected relative amplitude.
1	10°	0	—	—	0
2	18°	0·8	27	1·11	0·9
3	32°	1·2	27	1·21	1·5
4	46°	1·3	27·5	1·15	1·5
5	60·5°	1·25	[27·5]	1·15	1·4
6	75°	4·0	27·1	1·24	5·0
7	89°	4·6	27·6	1·13	5·2
8	105°	6·6	27·1	1·24	8·2
9	120°	6·6	27·0	1·26	8·3
10	134°	4·8	27·1	1·24	6·0
11	149°	3·3	26·9	1·29	4·3
12	162°	2·4	27·1	1·24	3·0
13	170°	2·0	[27·5]	1·15	2·3

It will be seen that the frequencies recorded in the fourth column are very uniform. In the case of the records 5 and 13 there were mishaps to the high-speed recorder, and no frequency was observed. The mean of the observed frequencies (27·5 ~) was therefore adopted in default of more accurate information, and is shown in square brackets in the table.

The observed relative amplitudes are plotted on a polar diagram in fig. 1. The line $\beta=0°$ is along the axis of rotation of the airscrew and in the direction in which the aeroplane would move if it were in flight.

The curve shows that there is a tendency for the sound to die away both forward and backward along the axis of

Fig. 1.



Amplitude of the fundamental sound of rotation in various directions from a two-bladed airscrew making 13·5 rev./sec.

rotation, and that there is a direction of maximum amplitude between $\beta=105°$ and $\beta=120°$.

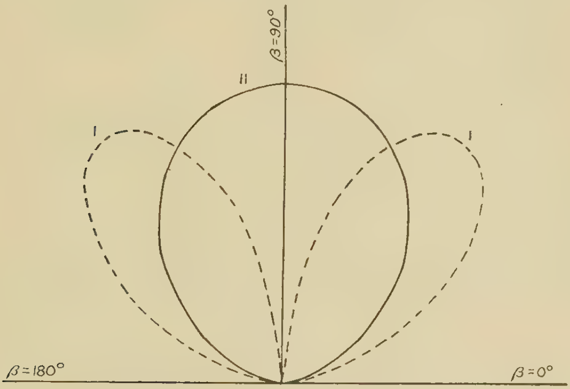
The irregularity which occurs between $\beta=0°$ and $\beta=90°$ was noticed in other sets of observations, but the exact shape was not repeated when the aeroplane occupied a different position relative to the microphone, and it seemed possible that reflexion of sound from the ground, or from parts of the machine, might be wholly or partly responsible for this effect. A maximum near the position A (fig. 1) occurred in all the observations, but not that at B, which seems to have been accidental. The true form of the curve is probably approximately along the broken line in fig. 1 *.

* This conclusion has been confirmed by subsequent experiments performed by Mr. C. F. B. Kemp in which a different method of sound-recording was employed.

§ 5. *Comparison of Observation with Theory.*

In fig. 2 the curves I and II were calculated from the theories of Lynam and Webb. The curve I corresponds to the first hypothesis and was obtained from the expression (2) of § 2, the following numerical values appropriate to the airscrew used in the experiment being employed: $m=2$, $\omega=13{\cdot}5 \sim = 27\pi$ radian/sec., $L=7{\cdot}4$ ft., $a=1100$ ft./sec. With the same values curve II, corresponding to their second hypothesis, was calculated from (4) of § 2.

Fig. 2.



Theoretical curves for sound distribution round airscrew. (Lynam and Webb.)

A comparison of the theoretical curves in fig. 2 with the experimental curve in fig. 1 brings out the following points:—

(i.) The experimental curve confirms the theoretical conclusion from both hypotheses of Lynam and Webb that the amount of sound projected along the axis of rotation tends to zero.

(ii.) The experimental curve confirms the conclusion of M. D. Hart * that the first hypothesis of Lynam and Webb is untenable because it gives zero sound-intensity in the plane of rotation.

(iii.) The agreement of the experimental curve with the sound distribution deduced from the second hypothesis of Lynam and Webb does not extend beyond that indicated

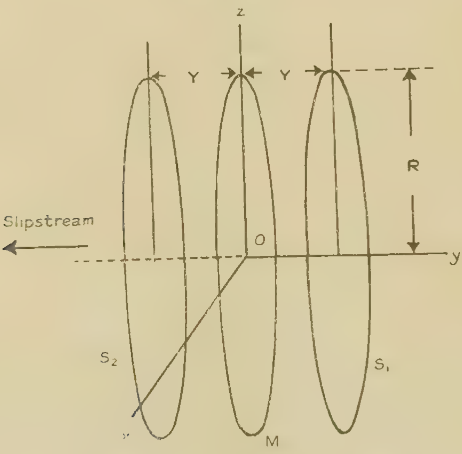
* Aero. Res. Comm., R. & M., No. 1310, p. 11 (1930).

in (i.), since the curve II in fig. 2 shows maximum amplitude in the plane of rotation, whereas the observations show maximum amplitude in a direction about 25° behind the plane of rotation.

(iv.) The experimental curve is not symmetrical about the plane of rotation, and hence the airscrew cannot be represented by any distribution of simple sources in the plane of rotation.

It must of course be borne in mind that the theory deals with the case of an airscrew in a homogeneous atmosphere free from reflecting surfaces, whereas in the experiments

Fig. 3.



Suggested arrangement of sources.

there must be some reflexion from the surface of the ground and from the engine and fuselage*.

The question arises whether a better representation of an airscrew as a source of sound can be arrived at by making use of the suggestion of Lynam and Webb that their two hypotheses might be combined.

In order to answer this question it is necessary first to refer to the expression for the velocity-potential at a distant point due to a ring of sources having the characteristic phase-relationship expected from the motion of the airscrew. In fig. 3 let xOz be the plane of rotation, the centre of the

* The wings, however, do not have any noticeable effect. This was shown by an experiment performed subsequently to that described in this note, in which a wingless fuselage was employed.

airscrew being at O. The potential due to a ring of sources of radius r with its centre on Oy at a distance y from O and in a plane parallel to xOz is proportional to *

$$J_m\left(\frac{m\omega r}{a}\sin\beta\right).\cos m\omega\left(t+\frac{y}{a}\cos\beta\right), \quad . \quad . \quad (9)$$

the notation being as in § 2. It is convenient to write k (equal to $2\pi/\lambda$, λ being the wave-length of the sound) for $m\omega/a$. Thus (9) may be written

$$J_m(kr\sin\beta)\,.\cos k(at+y\cos\beta). \quad . \quad . \quad . \quad (10)$$

An arrangement of sources that has been found by trial to give a distribution of sound resembling that observed in experiments consists of a ring of sources (S_1 in fig. 3) in front of the plane of rotation, a ring of "sinks" (S_2 in fig. 3) behind it, and a ring of sources M (fig. 3) in the plane of rotation. The sources M are intermediate in phase between those at S_1 and S_2. The distinction between a "source" and a "sink" is, of course, only one of phase, a "sink" lagging behind the corresponding "source" by half a period. If R is the radius of the rings S_1 and S_2, and Y the distance of their centres from O, the potential at a distant point of the doublets constituted by the sources and sinks S_1 and S_2 is, by (10), proportional to

$$J_m(kR\sin\beta)\left[\cos k(at+Y\cos\beta)+\cos\{k(at-Y\cos\beta)-\pi\}\right]$$
$$=-2J_m(kR\sin\beta)\,.\sin(kY\cos\beta)\,.\sin kat. \quad (11)$$

The potential of the ring of sources of intermediate phase is found by putting $y=0$ in (10) and retarding the phase by a quarter of a period. Also it will be assumed that $r=R$, as for the doublets. Thus the potential due to these sources is proportional to

$$J_m(kR\sin\beta)\,.\cos(kat-\pi/2)=J_m(kR\sin\beta)\,.\sin kat. \quad (12)$$

The total potential is found by addition of expressions proportional to (11) and (12). Thus (12) may be multiplied by a factor μ which will depend on the relative strengths of the sources in the plane of rotation compared with those composing the doublets, the strengths being equal when μ is unity. The total potential is therefore proportional to

$$J_m(kR\sin\beta)\{\mu-2\sin(kY\cos\beta)\}\,.\sin kat. \quad . \quad (13)$$

If the values of R and Y used by Lynam and Webb are

* Lynam and Webb, *loc. cit.* p. 6.

adopted, namely, $R=3L/4$ and $Y=3L/8$, and we assume $\mu=2$, the amplitude of the fundamental tone generated by the two-bladed airscrew used in the experiment would be proportional to

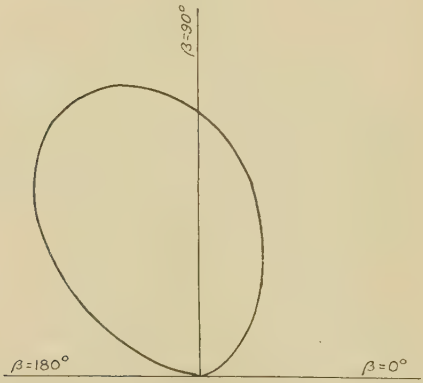
$$2J_2\left(\frac{3kL}{4}\sin\beta\right).\left\{1-\sin\left(\frac{3kL}{8}\cos\beta\right)\right\}, \quad . \quad (14)$$

where

$$L=7{\cdot}4 \text{ ft.} \quad \text{and} \quad k=m\omega/a=(2\times 13{\cdot}5\times 2\pi)\div 1100 \text{ ft.}^{-1}.$$

The distribution of sound round the airscrew corresponding to (14) is shown by the polar diagram in fig. 4, where

Fig. 4.



Distribution of sound round airscrew calculated from (13).

relative amplitude is plotted against β. The direction of maximum amplitude is when β is about 115° and agrees approximately with that found in the experiment (*cf.* fig. 1). The calculated distribution, however, shows no secondary maximum corresponding to that at A in fig. 1, and the amplitude for directions near $\beta=90°$ is relatively greater than that observed.

Better agreement with the observed distribution can be obtained by taking $Y=3L$ and $\mu=2{\cdot}6$. The amplitude in this case is proportional to

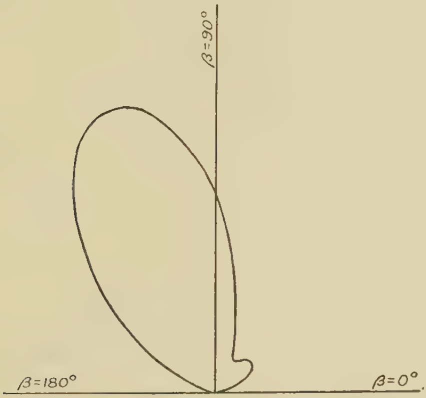
$$2J_2\left(\frac{3kL}{4}\sin\beta\right).\{1{\cdot}3-\sin(3kL\cos\beta)\}. \quad . \quad . \quad (15)$$

The corresponding polar curve is shown in fig. 5. It exhibits the main features of the observed curve (fig. 1), namely, maximum amplitude in a direction some 20° behind the plane of rotation, with a secondary maximum when β is about 45°. The observed amplitudes when β is greater than 135° are, however, considerably greater than those indicated by (15).

§ 6. *Summary and Conclusions.*

Experiment shows that the distribution of the fundamental sound of rotation round a two-bladed airscrew on a stationary

Fig. 5.



Distribution of sound round airscrew calculated from (15).

aeroplane is such that (i.) the sound tends to zero in directions forward—*i. e.*, against the slipstream—or backward along the axis of rotation, (ii.) there is a principal maximum in a direction about 25° behind the plane of rotation, and (iii.) there is a subsidiary maximum in front of this plane at abont 30° to 45° from the axis of rotation.

A suggestion made by Lynam and Webb has been used to construct a formula which gives results exhibiting the principal features of the observed sound distribution so far as the fundamental sound of rotation is concerned.

IX. *Classical Energy and the Interpretation of Schroedinger's ψ-function.* *By* A. Press *.

1. *Summary.*—It is an outstanding difficulty to the classicist to regard the total energy E of a system as negative, and likewise the potential energy V. Nevertheless the power and usefulness of the Schroedinger analysis is so extraordinary that even the most ardent classicist is struck with its uncanny power. In the following, the mechanics of the so-called asymmetrical diaphragm will be taken up in the ordinary way according to the ideas of Newton and the mechanists of the old school. Later a transformation will be employed that will lead to the Schroedinger type of equation. In a sense the loaded type of diaphragm contemplated can be regarded as analogous to the linear oscillator of Planck. It should be pointed out, however, that a radiation pressure term $\mathrm{R}dx/dt$ is here included. It will be found as a result that the ψ-function can be re-defined and led back, in its interpretation, to two components. The one is the quantity $\frac{1}{2}mv_s^2$, referring to a "steady state" value, and the other to the radiation energy $\int_0^{x_s} \mathrm{R}dx/dt.dx$ in going from a zero position of displacement to the amplitude value x_s characterizing the energy level E_s.

2. In a centrally-weighted diaphragm of the L. V. King type we have for the d'Alembertian force of reaction the expression

$$m \cdot \frac{d^2x}{dt^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.1)$$

The strain energy of the system can be written down as dependent on the function

$$\mathrm{F}_x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

which in general would not be constant, but rather a function of x. The radiation pressure, on the other hand, can be assumed given by the term

$$\mathrm{R} \cdot \frac{dx}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3)$$

whereas the three forces (2.1), (2.2), and (2.3) are to be understood as equated to an impressed force,

$$\mathrm{P}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.4)$$

* Communicated by the Author.

to satisfy the requirements of the conservation of energy principle.

Thus we have the fundamental equation

$$m \cdot \frac{d^2x}{dt^2} + F_x + R \cdot \frac{dx}{dt} = P. \quad . \quad . \quad . \quad (2.5)$$

It may be remarked that in the asymmetrical diaphragm it is usual to regard F as made up of terms after the following manner :

$$F_x = \alpha x + \beta x^2 + \gamma x^3. \quad . \quad . \quad . \quad . \quad (2.6)$$

Thus for harmonic variations of the time we would have, by way of example,

$x = x_0 + x_1$,

x_0 = constant, due to an L. V. King type of bias,

$x_1 = \Sigma_{n_1}(a_{n_1} + b_{n_1}j) \sin n_1\omega t.$ . . . . (2.7)

3. To obtain a better view of things the pressure equation (2.5) can be multiplied by the velocity $\frac{dx}{dt}$ of the diaphragm centre, so as to result in an activity equation. We then have that

$$m \cdot \frac{d^2x}{dt^2} \cdot \frac{dx}{dt} + F_x \cdot \frac{dx}{dt} + R \frac{dx}{dt} \cdot \frac{dx}{dt} = P \cdot \frac{dx}{dt} \quad . \quad (3.1)$$

Clearly if a time integration is to be resorted to then the impressed work, or energy supplied to the system, will be given by the equation,

$$\int_{t=t_0}^{t=t} P \cdot \frac{dx}{dt} \cdot dt = \int_0^{x=x} P \cdot dx. \quad . \quad . \quad . \quad (3.2)$$

The time variable dt is brought in first since the x's are functions of the time t.

For the force F_x, producing the strain or potential energy displacement x, it follows similarly

$$\int_{t_0}^{t} F_x \cdot \frac{dx}{dt} \cdot dt = \int_0^{x} F_x \cdot dx, \quad . \quad . \quad . \quad . \quad (3.3)$$

and for the remaining terms we need to write

$$\int_{t_0}^{t} \left(m \cdot \frac{d^2x}{dt^2} + R \cdot \frac{dx}{dt} \cdot\right) \frac{dx}{dt} \cdot dt = \int_0^{x} (\quad) dx. \quad . \quad (3.4)$$

In each of the cases (3.2), (3.3), and (3.4) a series of frequency terms must appear on both sides for the general problem of harmonic oscillations. If then the integrations

are to be performed at all it will be perfectly legitimate to regard the process of integration as applying to each particular frequency throughout in a separate manner. Thus, so far as the upper limit is concerned a "stationary" value x_s can be used which is discrete and characterizes the appropriate harmonic amplitude out of the infinitude of x-values possible and appropriate to (2.5). In this way therefore it is at once seen that

$$\int_0^{x_s} \mathrm{P} \,.\, dx = \mathrm{E}_s = \text{maximum or total energy delivered per cycle} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.5)$$

$$\int_0^{x_s} \mathrm{F} \,.\, dx = \mathrm{V}_s = \text{Total momentary strain energy (potential) occurring when } \mathrm{E}_s \text{ attains its } x_s\text{-value, out of all the possible } x\text{-values characterizing the equation (2.5).} \quad . \quad . \quad . \quad . \quad . \quad (3.6)$$

It is justified therefore to write instead of (3.1) that

$$\int_0^{x_s} \left(m \,.\, \frac{d^2x}{dt^2} + \mathrm{R}\frac{dx}{dt}\right) dx - (\mathrm{E}_s - \mathrm{V}_s) = 0. \quad . \quad (3.7)$$

This condition must be satisfied for all of the infinitude of x_s's occurring in the gamut or continuum of x.

4. The Schroedinger type of equation is now easily arrived at ; for if a wave-form of equation is desired for a definite "stationary" frequency $\nu_s = \frac{\omega_s}{2\pi}$, it will be sufficient to define a function ψ, such that in effect

$$\int_0^{x_s} \left(m \,.\, \frac{d^2x}{dt^2} + \mathrm{R}\frac{dx}{dt}\right) dx = \frac{h^2}{8\pi^2 m} \,.\, \frac{1}{\psi} \,.\, \frac{d^2\psi_s}{dx_s^2}. \quad (4.1)$$

In view of (3.7) we have at once that

$$\frac{d^2\psi_s}{dx_s^2} - \frac{8\pi^2 m}{h^2}(\mathrm{E}_s - \mathrm{V}_s) \,.\, \psi_s = 0. \quad . \quad . \quad . \quad (4.2)$$

The latter is Schroedinger's equation except for the minus sign in front of the bracketed expression $(\mathrm{E}_s - \mathrm{V}_s)$.

To some extent (4.2) offers an improved visualization of the radiation problem. Thus we can write

$$\mathrm{V}_s = \frac{e^2}{r_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.3)$$

without introducing a disconcerting minus sign. The same sort of thing is true of E_s which will be shown later is a constant, and therefore independent of x_s for any s-state.

This would not be true for the potential function V_s, which depends on the size of x_s (or rather r) according to (4.3). Such is the significance of the really parametral type of equation first brought into radiation theory by Schroedinger.

5. It is possible to give a physical meaning to the integral shown in equations (3.7) and (4.1). Thus we have that

$$\int_0^{x_s} m \cdot \frac{d^2x}{dt^2} \cdot dx = m \int_0^{x_s} \frac{dv}{dt} \cdot \frac{dx}{dt} \cdot dt$$

$$= m \cdot \int_0^{x_s} \frac{d}{dv}(v^2)\, dv = \tfrac{1}{2} \cdot m v_s^2 . \quad . \quad (5.1)$$

In general therefore it is to be presumed that v_s will be a function of x_s and therefore ψ also. At the same time we note that

$$\int_0^{x_s} \left(\mathrm{R} \cdot \frac{dx}{dt}\right) dx = w_s, \quad . \quad . \quad . \quad . \quad (5.2)$$

where w_s expresses the energy radiated away during the s-state displacement from $x=0$ to $x=x_s$. From the nature of the equation

$$\frac{d^2\psi_s}{dx_s^2} - \frac{8\pi^2 m}{h^2}(\mathrm{E}_s - \mathrm{V}_s) \cdot \psi_s = 0 \quad . \quad . \quad . \quad (4.2)$$

it would seem that, together with ψ_s, both w_s and v_s are functions of x_s. Yet according to (3.7) ψ_s is essentially a function of v_s and w.

An operator method of solving the last-mentioned equation has already been given, leading to the Balmer series (see paper by writer in the Phil. Mag., July 1928, pp. 33–48).

June 1931.

X. *Additional Experiments on Moving-Coil Reproducers and on Flexible Disks. By* N. W. McLachlan, *D.Sc., M.I.E.E., F.Inst.P.**

[Plates I.–V.]

Synopsis.

1. The Modulus of Elasticity of Paper (E).
2. Stiffness of a Conical Sheet.
3. Influence of Reinforcing Diaphragm of Moving-coil Reproducer.
4. Combination Modes of Reed-driven Paper Disk.

* Communicated by the Author.

ABSTRACT.

In this paper various details of moving-coil reproducers are discussed. The modulus of elasticity of paper is determined and used to compare the stiffness of conical sheets and disks of the same radius. A series of experiments is outlined, showing the influence of reinforcing certain portions of the diaphragm. The combination modes of a reed-driven paper disk are treated, and by comparison with theory, heterogeneity of the paper is disclosed. The exact solution of a lossless reed-driven disk vibrating *in vacuo* is given, due allowance being made for variation in the apparent mass of the reed itself. The case of a coil-driven circular aluminium disk is examined experimentally, and the frequency band occupied by the first symmetrical mode is shown to be only 5 cycles. With constant current in the coil, the output at resonance is some 2000 times greater than at non-resonant frequencies. The influence of the magnetic field upon the output and upon the coil impedance of a reproducer is analysed. The output criterion of the magnet is stipulated in terms of total gap area, air-gap flux density, and the coil space factor. It is shown that the oscillation of a coil driven by an alternating current in a non-uniform magnetic field is accompanied by a motion of translation. This is akin to the process of rectification.

Lastly, a series of impulse records showing the natural damped oscillations of moving-coil reproducers of the horn and hornless (large diaphragm) variety are given. From these records it is deduced that their field of utility lies in the determination of the main symmetrical mode in the upper register and the low frequency modes associated with the surround.

---

1. *The Modulus of Elasticity of Paper* (E).

In the course of experiments described in a former contribution *, it was necessary to find the modulus of elasticity E for certain grades of paper. The method

* Other properties of the disks and diaphragms treated herein will be found in Phil. Mag. xii. p. 771 (1931) and Proc. Phys. Soc. xliv. (Jan. 1932).

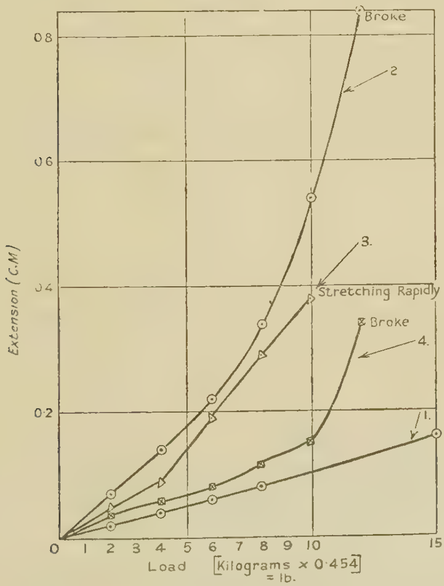
of procedure was the customary one of taking readings of load and extension. The results of some of the tests are

TABLE I.

Showing Young's Modulus of Elasticity (E) for paper at 20° C. Test length 50 cm. [For steel $E = 2 \times 10^{12}$ dynes cm.$^{-2}$.]

Type of paper.	Thickness (cm.).	Mass per unit area (gm. cm.$^{-2}$).	Density. (gm. cm.$^{-3}$).	E dynes cm.$^{-2}$.
A	$1{\cdot}5 \times 10^{-2}$	$1{\cdot}0 \times 10^{-2}$	$6{\cdot}7 \times 10^{-1}$	$2{\cdot}0 \times 10^{10}$
B	$2{\cdot}1 \times 10^{-2}$	$1{\cdot}4 \times 10^{-2}$	6·6	$1{\cdot}9 \times 10^{10}$
C	$4{\cdot}0 \times 10^{-2}$	$2{\cdot}5 \times 10^{-2}$	6·0	$1{\cdot}9 \times 10^{10}$
D	$2{\cdot}3 \times 10^{-2}$	$1{\cdot}8 \times 10^{-2}$	7·8	$4{\cdot}6 \times 10^{10}$

Fig. 1.



Load-extension curves for paper strips, width 2·54 cm., length 50 cm., thickness (1) $= 4 \times 10^{-2}$ cm., (2) $= 1{\cdot}5 \times 10^{-2}$ cm., (3) $= 2{\cdot}1 \times 10^{-2}$ cm., (4) $= 2{\cdot}3 \times 10^{-2}$ cm.

shown in Table I. In a number of cases the relationship between load and extension was substantially linear within definite limits. Curve 1, fig. 1, illustrates a typical case. By testing more than one sample of the paper it was

found that E varied, as shown in curves 3 and 4, the load/extension relationship was not always linear, whilst in some cases the paper stretched abnormally, as shown by curve 2, fig. 1. Also, the test specimen did not always return to its original length after removal of the load. Owing to variations in E, a paper cone will not act as a homogeneous sheet, so that peculiarities in nodal figures and variation in the frequencies of the modes is to be expected. Moreover, a large output from a paper diaphragm might be accompanied by harmonics due to inelasticity, especially if the wrong kind of paper or a faulty piece were used.

When the paper was baked in an oven for several hours at 110° C. the values of E were raised considerably. For example, one grade of paper in the normal state gave $E=1{\cdot}9\times10^{10}$ dynes cm.$^{-2}$, whilst after baking it was $3{\cdot}4\times10^{10}$ dynes cm.$^{-2}$. The latter operation was essential in making coils, since the paper formers on which they were wound had to be baked complete with the coils. For any type of paper the value of E varies with the sample and with the humidity. Moreover, the tabular data must be regarded as average values indicative of what is to be expected. In making calculations relating to a certain class of paper, it is advisable to measure the value of E at the time of the experiment rather than extract it from a table.

2. *Stiffness of a Conical Sheet.*

When a mechanical system simulates a simple loaded coil spring, both the dynamical and statical stiffness coefficients are identical, the former being defined by the relationship $k_1=\omega^2 m$, where m is the mass and $\frac{\omega}{2\pi}$ the resonance frequency. This argument is valid for the simple longitudinal or torsional oscillations of a uniform bar. Where flexible disks or conical sheets are concerned, the dynamical and statical stiffness coefficients are no longer identical owing to the totally different physical conditions in the two cases.

Since it is impracticable to define the stiffness of a flexible disk on these lines, the same argument applies to a conical sheet. Moreover, other means must be sought to convey the idea of stiffness. It is proposed to find the thickness of a disk of equal radius whose first centre-moving symmetrical mode occurs at the same frequency as that of the conical sheet of like material. This must be regarded as merely illustrative, since the stresses in the two cases are of a

different nature *. The following data apply to a free-edge conical diaphragm (fig. 10 of the former paper).

Table II.

Thickness of paper	$=5\times 10^{-2}$ cm.
Radius of cone	$=16{\cdot}7$ cm.
Apical angle	$=160°$.
First symmetrical mode	$=350$ cycles per second.
Second symmetrical mode	$=664$,, ,, ,,

The frequency of the first mode of a paper disk equal in radius and in thickness to that of the cone is 22 cycles per second.

From Warren's analysis †

$$h^4 = bg$$
$$= 2\pi\omega^2 q \cdot \frac{6}{\pi t^3} \cdot \frac{\sigma^2-1}{\sigma^2} \cdot \mathrm{E}.$$

Taking the radius a as constant, then since q the mass of the disk per unit area varies as the thickness t, we get $(ha)^4\mathrm{D} = \omega^2/t^2$, where D is a constant, or $t = \mathrm{D}_1\omega/(ha)^2$. For any given value of ha corresponding to a mode, the frequency increases directly as the thickness of the disk. Applying this result to the preceding data, we find that the thickness of a disk whose first centre-moving symmetrical mode occurs at 350 cycles per second is $\frac{350}{22}\times 5\times 10^{-2} = 0{\cdot}8$ cm., and its mass is about 15 times that of the cone. Moreover, from this view-point, the cone is equivalent to a disk $\frac{350}{22} = 16$ times as thick as the sheet of which it is constructed. The large degree of stiffness concomitant with a conical shape needs no comment.

The second symmetrical mode of the above cone occurs at 664 cycles per second, whilst that of the disk is 1380 cycles per second. For this mode, the disk 0·8 cm. thick is $\left(\frac{1380}{664}\right)^2 = 4{\cdot}5$ times stiffer than the cone. In other words, under our method of computation the cone is not so stiff at its second symmetrical mode as a disk 0·8 cm. thick. This result follows from the different nature of the stresses in the two cases.

* Phil. Mag. xii. p. 771 (1931). No allowance has been made for the mass of the coil.

† Phil. Mag. ix. pp. 881–901 (1930).

A second example of a diaphragm with a smaller apical angle is given in Table III. below.

TABLE III.

Thickness of paper	$=2{\cdot}1\times10^{-2}$ cm.
Radius of cone..............................	$=12$ cm.
Apical angle	$=90^{\circ}$.
E (Young's Modulus)	$=1{\cdot}9\times10^{10}$ dynes cm.$^{-2}$.
First symmetrical mode	$\doteq$ 900 cycles per second.

The first mode of a paper disk of equal radius is 11 cycles per second. Thus the thickness of disk having a fundamental frequency of 900 cycles is $\frac{900}{11}\times2{\cdot}34\times10^{-2}=1{\cdot}9$ cm., or about 57 times the mass of the cone and thrice the thickness of the equivalent disk in the preceding example. Although the grades of paper are different in Tables II. and III. these calculations show the marked increase in stiffness with decrease in apical angle.

Having given examples of conical stiffness for symmetrical modes, we now turn our attention to radial modes. The first radial mode* of a circular disk whose properties are given in the second row of Table I., occurs at 11 cycles per second, whereas that of a 90° cone, 12 cm. radius, occurs at 55 cycles per second. In this respect, the cone is equivalent to a disk of equal radius, but 55/11 = 5 times as thick.

These examples are adequate to illustrate the enormous gain in stiffness which accrues from the use of a conical sheet as compared with a flat disk.

3. *Experiments showing the Effects of modifying the Diaphragm Structure of a Moving-coil Reproducer.*

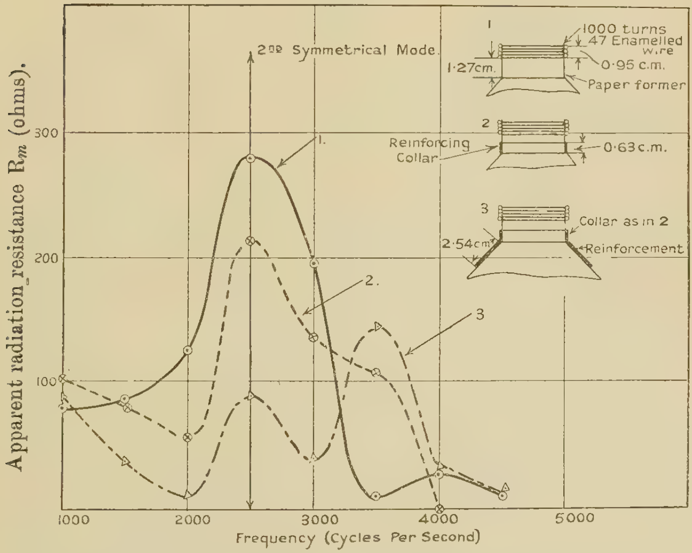
In this section it is proposed to show broadly the influence of reinforcing various parts of the diaphragm structure of a reproducer. The paper was coated with aeroplane dope on each side. Only a few points were taken on the curves to show the general behaviour in each case. Moreover, some of the lesser irregularities in the curves may have been missed.

In fig. 2, curve 1, we have the case of a doped diaphragm supported on an annular rubber surround. The coil consisted of 1000 turns of 47 S.W.G. enamelled wire wound on a paper

* Four radial nodes or two diameters.

former and bakelized. The free length of the latter was 1·27 cm. Here the main resonance occurs at 2500 cycles. Curve 2, fig. 2, shows the effect of reinforcing the paper coil former (neck) near the diaphragm with a paper collar 0·63 cm. wide. The main resonance still remains at 2500 cycles. Its magnitude is reduced somewhat, whilst the output just above 3000 cycles is increased. Curve 3, fig. 2, shows a transformation due to a strip of paper 2·54 cm. wide, glued round the top of the cone adjacent to the collar. The

Fig. 2.



Apparent radiation resistance of coil-driven diaphragm under different mechanical conditions.

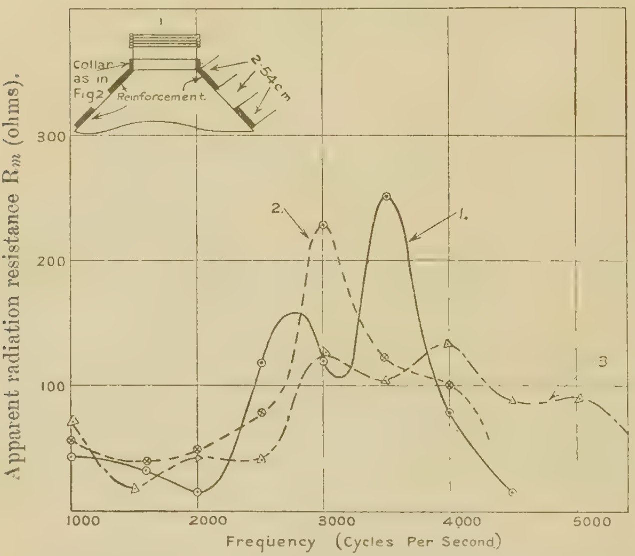
2500 cycle resonance persists although reduced in intensity, whilst a more powerful resonance appears at 3500 cycles.

Curve 1, fig. 3, shows what happened when a second reinforcing ring was added to the diaphragm 2·54 cm. from that used for curve 3, fig. 2. The 2500 cycle resonance is still existent, but is overshadowed by that at 3500. The effect of removing the collar on the coil former and using half the original free length is shown in curve 2, fig. 3; the other conditions are as before. The result is to move the resonance up to 3000 cycles. When large quantities of seccotine were applied to the junction of the coil and cone

and allowed to set hard, the result is exhibited in curve 3, fig. 3. There are signs of a resonance at 2000 cycles, that at 3000 cycles remains, but others are introduced at 4000 and 5000 cycles.

By way of contrast the curve of fig. 4 is for a doped diaphragm of identical dimensions but with a free former length of only 0·3 cm., there being no reinforcement with seccotine. The addition of reinforcement to the diaphragm does not improve the reproduction, although it may extend

Fig. 3.



(1) See sketch inset. (2) Conditions as for (1), but collar removed and free length of former 0·48 cm.; (3) as in (2), but with more seccotine round joint to cone.

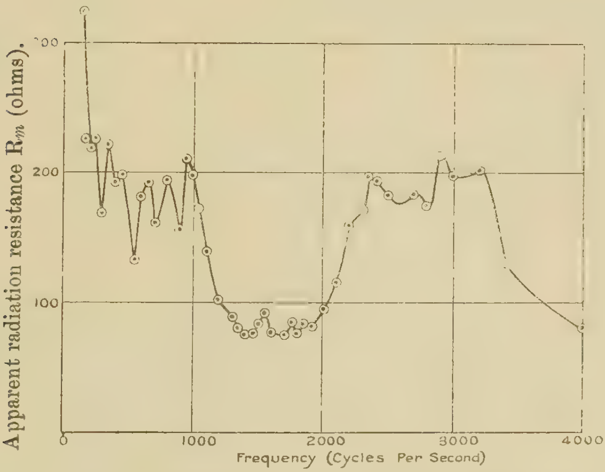
the upper register to a degree. As a general rule, whatever artifices are adopted, the output is rapidly attenuated above 6000 cycles, and other means have to be utilized to obtain a reasonable output up to 10,000 cycles. The importance of extending the register as far as 15,000 cycles is not generally realized. The average system does not properly reproduce hand-clapping, speech, jingling of keys or coins, and transients in general, owing to attenuation of the upper frequencies. The natural frequencies of coins are of the order 10,000 cycles or more, according to the diameter. Hence

the reproducing system must be sensitive to frequencies of this order *.

4. *Combination Modes of Reed-driven Free-edge Circular Paper Disk.*

Knowing the values of E, q, and σ for paper, it is possible to calculate the *in vacuo* effective or apparent mass curves on the assumption that the paper is homogeneous and free from loss. By superposing on these the curve for the reed, as shown in a previous paper †, the frequencies of the

Fig. 4.



Apparent radiation resistance of standard doped diaphragm with 1200 turn-coil and rubber surround. Free length of coil former about 0·3 cm., there being plenty of seccotine on the joint to the cone. Main symmetrical mode from impulse test is in the neighbourhood of 2600 cycles per second as shown in fig. 18.

combination modes can at once be determined. Comparison with actual results reveals the idiosyncrasies of the paper and the influence of losses and accession to inertia.

A disk of crisp paper was mounted centrally on the reed of a "Lion" ‡ unit and securely held between two circular metal washers 0·8 cm. radius. Starting at 50 cycles, current was supplied to the unit and sand patterns were

* See 'Wireless World & Radio Review' (April 3, 1929).
† Phil. Mag. xi. p. 13 (Jan. 1931).
‡ Phil. Mag. xi. p. 8 (1931).

observed up to 1300 cycles per second. The results are indicated in fig. 5. The frequencies marked on the horizontal axes were calculated, and the M_a curves merely *sketched* in to show the trend of the results. The numbers on the reed curve were the frequencies obtained experimentally. With the exception of the second and fourth modes at 125 and 610 cycles respectively, they fit in fairly well. The frequencies at these modes are higher than one would expect theoretically, and this points to heterogeneity

Fig. 5.

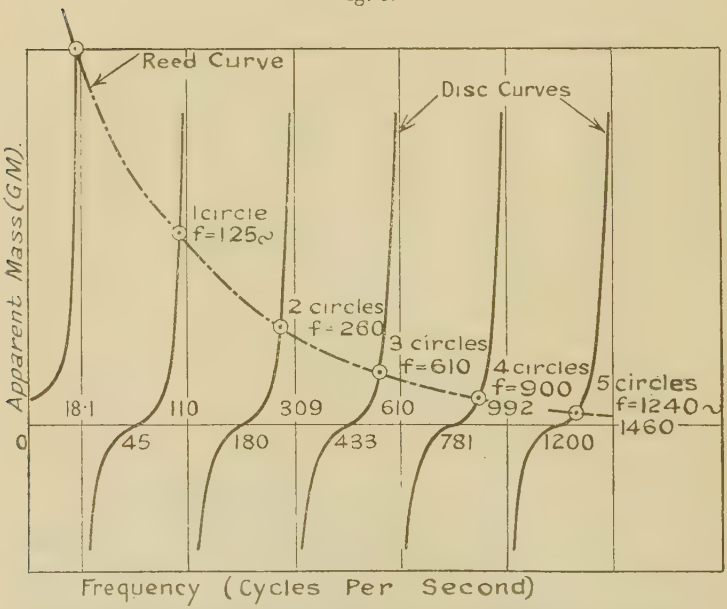


Diagram illustrating combination modes of reed-driven paper disk

of the paper. Although the modes are marked 1, 2, 3, etc. circles, the sand-patterns are by no means as simple as this. Some of the nodal figures are illustrated in fig. 6. Patterns sometimes occur when they are not expected, which points to heterogeneity of the paper. No semblance of regularity was seen until, at 1240 cycles, portions of five concentric circles appeared. Owing to transmission loss the centre-stationary modes were too obscure to be obtained with certainty, and only the centre-moving modes were recorded. The loss may partly account for the absence of regular sand figures. If the effective mass curves for the disk fall below the reed

at certain frequencies no marked resonance occurs. (See Phys. Soc. paper for influence of loss on M_e curves.)

An impulse record of the combination was taken *, and it is reproduced in fig. 7 (Pl. I.). The main frequencies are 260 and 1200 cycles per second, which correspond closely to the third and sixth modes found by sand figures. As in the case of conical diaphragms, there are two frequencies, one low, the other high, prominent on such records. Evidently the combined effect of magnetic and frictional damping is adequate to reduce the other oscillations considerably.

Fig. 6.

Nodal patterns for reed-driven paper-disk 10 cm. radius, 5×10^{-2} cm. thick. As f increases from 60~, the sand wanders over the surface like a snake. This effect has also been encountered in thin metal disks at low frequencies. The sand appears to flow like a fluid confined to a definite channel. At 90~there was one clearly defined diameter.

(*a*) Not a symmetrical mode; probably due to heterogeneity of paper.

(*b*) Corresponds to one circle on a homogeneous disk.

(*c*) Corresponds to two circles (see fig. 7) on a homogeneous disk

(*d*) Corresponds to three circles on a homogeneous disk.

(*e*) Corresponds to five circles (see fig. 7) on a homogeneous disk.

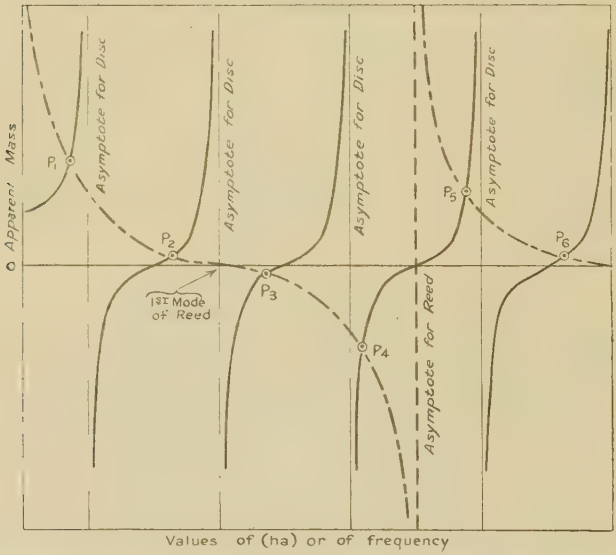
---

* Phil. Mag. xi. p. 49 (Jan. 1931).

5. *Exact Solution of Combination Modes of Lossless Reed-driven disk* in vacuo.

The solution indicated in fig. 5 of the present paper and in fig. 4 of the former paper is only correct if the reed can be treated as a simple coil spring whose apparent mass is invariable. In our case, apart from losses and accession to inertia, the results are fairly accurate for three reasons: (1) the restoring force on the reed was supplied by a torsional member, thus preserving the coil-spring analogy; (2) the modes were well below the natural frequency of the reed itself (not the torsional member); and (3) the accesssion to inertia was relatively small. If, however, an ordinary reed had been used, whose first mode occurred at, say, 700 cycles, the results at the upper frequencies would have been in error. In fig. 5 the reed curve is ultimately asymptotic to a horizontal axis. Actually the reed should be represented by a family of curves similar to those for the disk, since it has an infinite number of modes.

Fig. 8.



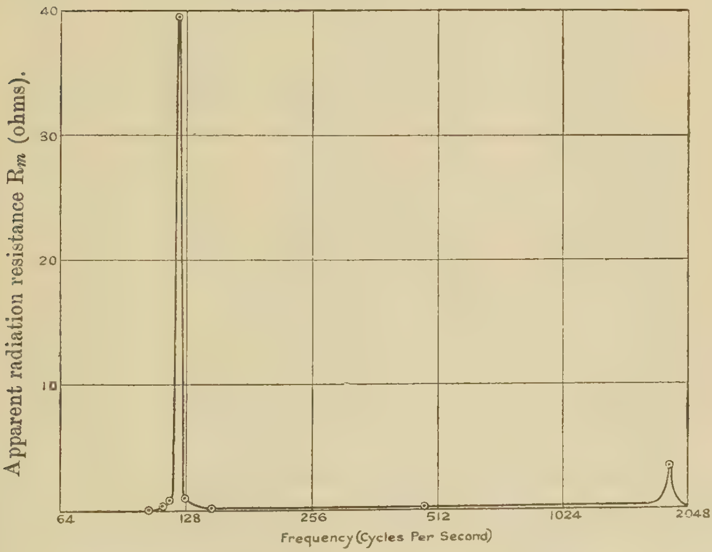
Curves (not calculated) illustrating the exact solution of the combination modes of a cantilever reed and a circular disk. It assumed that frictional and other losses are absent and the system is *in vacuo*.

If M_d and M_r are the apparent masses of the disk and reed respectively, the modes of the combination are given

by $M_d - M_r = 0$. Whence the solution is found by plotting M_d in the ordinary way on an *ha* base, whilst M_r is *inverted* and plotted on the same base, as shown in fig. 8. The abscissæ of the points of intersection of the two sets of curves provide the necessary values of frequency.

The appropriate curves are calculated from Warren's formulæ *. He gives the case of a disk with free and with fixed edge so that both can be treated. The cantilever reed case with a harmonic force applied to the free end is

Fig. 9.



Apparent radiation resistance of freely suspended coil-driven aluminium disk 10 cm. radius, 0·055 cm. thick, coil 2·5 cm. radius having 40 turns of 28 S.W.G.

not given, but can be deduced from expressions (6) and (7) with the conditions $D^2y=0$ at $x=0$ (free end), $Dy=0$ and $y=0$ at $x=l$ (fixed end).

Substituting in (6) and (7), we obtain the apparent mass of the reed

$$M_r = \frac{m}{\gamma} \frac{(1+\cosh\gamma x\cos\gamma x)}{(\sinh\gamma x\cos\gamma x - \cosh\gamma x\sin\gamma x)},$$

* Phil. Mag. xi. p. 883 (May 1930).

where m = mass of reed per unit length (assumed uniform).

$\gamma = \mu\omega^2 m.$

$\mu = 1/EI.$

E = Young's Modulus.

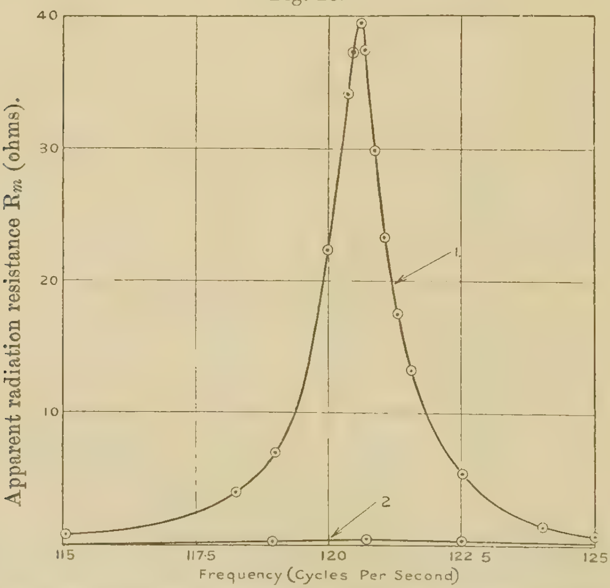
I = Moment of Inertia about neutral axis.

$\omega = 2\pi \times$ frequency.

6. *Modes of a Coil-driven Free-edge Circular Aluminium Disk.*

A series of experiments was conducted on a thin aluminium disk driven concentrically by a bakelized 40 turn

Fig. 10.



Curve for coil-driven aluminium disk at first symmetrical mode (1) compared with that of radial mode of paper cone (2).

coil of 28 S.W.G. Using a long paper former and an elastic thread suspension, the disk was kept in a horizontal plane 2·5 cm. from the face of the electro-magnet so that eddy-current damping due to vibration in the leakage field was small.

The data obtained by measurement of the apparent radiation resistance (field off and on) are illustrated in fig. 9 and Table IV. An enormous resonance occurs at 120·6 cycles, which in appearance resembles the selectivity curves

of modern radio circuits. To exhibit this to greater advantage, it has been plotted on the same diagram (fig. 10) as a radial mode curve for a free edge paper cone. The much greater damping of the paper is indicated by the grotesque comparison of the two curves. R_m for the disk is nearly 40 ohms, whilst that for the cone is only 0·24 ohm. In extenuation, it should be mentioned that the mode of the disk is symmetrical, whilst that of the cone is radial. This partly accounts for the large difference in R_m. Accurate location of the resonance at 120·6 cycles necessitated bridge

Table IV.

Modes of Coil-driven Free-edge Circular Aluminium Disk. Radius = 10 cm., thickness = 0·055 cm., mass = 47 gm. Mass of coil and former = 7·84 gm.

Frequency (cycles per second).	Nodal pattern.	R_m apparent radiation resistance (ohms).
120·6	One circle, broken twice (*a*).	**39·5**
200	Tendency for a curved diameter.	0·06
484	Two circles (*e*).	**3·04**
700	Indefinite.	0·145
950	,,	0·055
1850	Three circles outside and one inside coil.	**3·31**

measurements to 0·1 cycle per second, or about 1 part in 1000. On a percentage basis this resonance is not nearly so sharp as that of a good radio circuit.

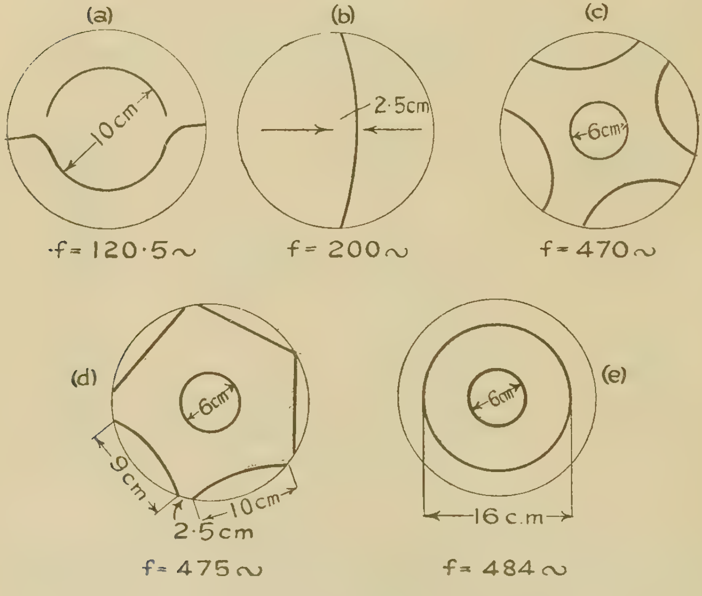
The nodal pattern for the first mode is shown in fig. 11 (*a*). It is an approximation to a circle 5 cm. radius, and the incompleteness at the sides is probably due to a tendency for two radii, although it may be caused by variations in the mechanical properties of the aluminium sheet. The resonance at 120·6 cycles is in fair agreement with the calculated value, due allowance being made for the mass of the coil and the portion of the disk within it.

A minor mode occurs at 200 cycles, the corresponding nodal figure being sketched in fig. 11 (*b*). The second centre-moving symmetrical mode occurs at 484 cycles, but its

magnitude is only 8 per cent. of the first mode. The nodal pattern, fig. 11 (*e*), consists of two concentric circles, but these were irregular. At 475 cycles the outer portion of the pattern resembles a pentagon, and it is reproduced in fig. 11 (*d*). A few cycles below this it alters to fig. 11 (*c*), which manifests a tendency for radii to be formed. On the whole, the nodal patterns are rather hectic, and remind one of those pertaining to paper disks

The next resonance of importance occurs at 1850 cycles, there being three circles outside and one inside the coil. The

Fig. 11.



Nodal patterns for coil-driven disk of fig. 9.

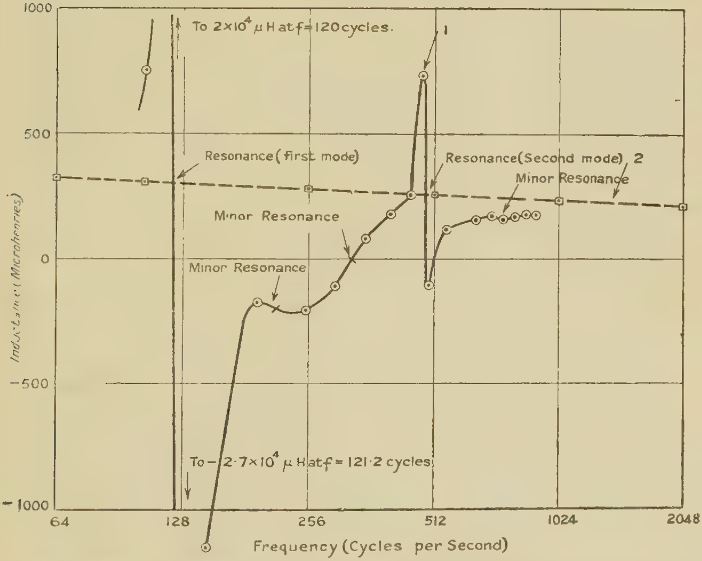
circles were more clearly defined than at lower frequencies, an effect also observed with paper disks. The absence of a prominent resonance between 484 and 1850 cycles is due to the influence of the portion of the disk within the coil upsetting the phase relationship in the disk *.

There were other modes of negligible importance yielding indefinite nodal patterns. The centre fixed modes were hardly detectable, excepting from the effective mass curve *.

* Proc. Phys. Soc. *loc. cit.*

Experiments with an aluminium disk three times as thick (0·165 cm.) used in previous work, showed that the modes were not revealed so readily as those with a central reed drive. The disk could, of course, be centrally driven by attaching a small cone or a conical spider between it and the coil. It is rather a coincidence that the 1850 cycle resonance occurs in the neighbourhood of those found with 90° conical paper diaphragms about 12 to 14 cm. radius. The magnitude of R_m for the disk is about ten times that for a paper diaphragm.

Fig. 12.



Curves showing variation in inductance of coil driving aluminium disk (1) coil free L_1, (2) coil fixed L_0.

At 120·6 cycles the resistance of the coil when fixed is about 1·1 ohm, but R_m, the added resistance due to motion, is 39·5 ohms, *i. e.*, some 36 times greater. Thus the apparent radiation efficiency is $\frac{39·5}{40·6} \equiv 97$ per cent., which is surprisingly high *. At 250 cycles R_m is only 0·02 ohm, corresponding to an efficiency of 2 per cent. The ratio of the values of R_m at these frequencies is $\frac{39·5}{0·02} = 1975$,

* No baffle was used.

which demonstrates quite forcibly the increase in output due to resonance. The inductance curve of the coil "free" (L_1) is of considerable importance, since it indicates variations in the effective mass of the system *. It is plotted in fig. 12. Peculiarities in curvature correspond to resonance and the relative changes in L_1 are more pronounced than the corresponding change in resistance R_1.

7. *Influence of Magnetic Field Strength in Moving-coil Reproducer.*

From an analytical viewpoint, the mechanical and electrical systems in a moving-coil reproducer are interlinked by the quantity $C^2=(2\pi rnH)^2$. This can conveniently be regarded as the electro-mechanical conversion factor, and it depends upon H, the strength of the magnetic field. Moreover, the influence of the latter can be predicted by examination of the various formulæ which contain C^2, provided the air-gap length and the dimensions of the moving coil remain constant.

In the absence of elastic constraint, the pure radiation resistance

$$R_a=C^2\cos^2\frac{\theta}{B}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)\dagger$$

where

$$\cos\theta=\frac{B}{(B^2+\omega^2 m^2)^{1/2}}.$$

Now B and m are independent of electrical variations since they pertain entirely to the mechanical system. Thus R_a varies directly as H^2, and therefore a strong magnetic field is desirable to enhance the sound output.

The motional capacity is given by

$$C_m=\frac{m}{C^2(1-\cos^2\theta)}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$\mathrm{Cos}^2\,\theta$ is usually small compared with unity, and is not influenced by H. Thus C_m varies inversely as H^2. Moreover, a large value of H is accompanied by a small value of C_m, and therefore by increased reactance which causes a reduction in low frequency current. In practical reproducers this is useful in limiting resonance effects when the power valve is a triode (low resistance). If a pentode is used, its resistance

* Proc. Phys. Soc. *loc. cit.*

† Phil. Mag. vii. p. 1017 (1929). R_m includes losses, but R_a is due to sound radiation alone.

is so high that the current is substantially constant over a wide frequency band, and the influence of the increased magnetic field in reducing resonances is negligible. If the edge of the diaphragm is reinforced and there is no surround, the lower register is relatively weak. A very strong magnetic field will reduce it still further if the power valve is a low resistance triode.

Hitherto we have discussed the influence of H without reference to the area of the air-gap or the space available for the moving coil. When the overall dimensions of an electromagnet are restricted owing to practical and economical considerations, a limit is set to the heat dissipation and therefore to the ampere turns or total magnetization. The maximum possible flux density in the gap is obtained when the whole magnetization is utilized there, *i. e.*, the steel path reluctance is zero (permeability $=\infty$) and there is no leakage. Under this condition $\mathrm{H}=1{\cdot}257\,\frac{ni}{l}$, where ni = total ampere turns and l = length of gap. Assuming $ni=3000$ and $l=0{\cdot}16$ cm., H = 23,600 lines per square centimetre. This approaches double the value in an actual case, and indicates the influence of the magnetic reluctance of the steel. Further information in this direction will be found in a former paper on magnetic measurements*.

We can approach the problem in a different manner. Owing to magnetic leakage within and without the magnet, experimental observation shows the total flux half-way down the central pole to be about 1·65 times that in the air-gap. Assuming the greatest flux density in the central iron pole to be 16,000 lines cm.$^{-2}$, the greatest air-gap density (mean over the axial length) †, is

$$\mathrm{H}=\frac{16{,}000}{1{\cdot}65}\times\frac{\text{cross-sectional area of central pole}}{\text{mean area of annular gap}}$$

(see fig. 13)

$$=\frac{\pi r^2}{2\pi r b}\,9{\cdot}7\times10^3=\frac{r}{2b}\,9{\cdot}7\times10^3.$$

Taking $r=2{\cdot}5$ cm., $b=0{\cdot}95$ cm., then H = 12,800 lines cm.$^{-2}$ is the greatest possible air-gap flux density on the above hypothesis. The ratio $r/2b$ cannot be increased indefinitely by reducing b, since the pole tips would be saturated, causing increased reluctance and leakage. These data agree with

* 'Wireless World,' p. 600 (Nov. 26th, 1930).

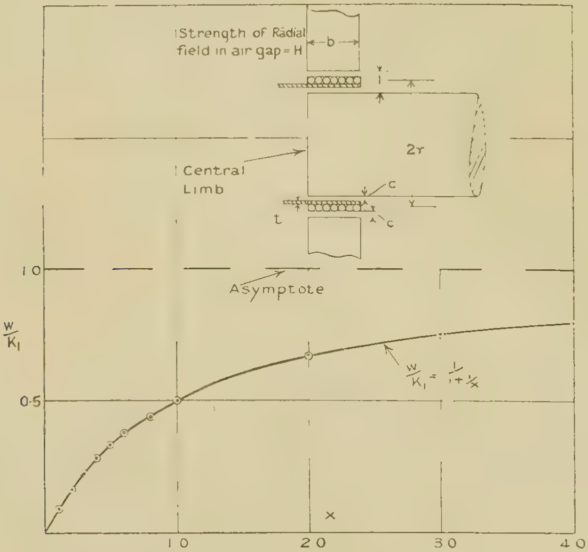
† *Loc. cit.*

measurements on the electromagnet cited in the previous paper, which shows the argument to have some value, although it is admittedly only a cursory survey of the problem whose accurate solution is very complex.

In the preceding case we can show the influence of reluctance and leakage numerically. The ampere turns for the air-gap alone are

$$\frac{Hl}{1\cdot257} = \frac{12{,}800 \times 0\cdot16}{1\cdot257} \doteqdot 1630 \text{ or}$$

Fig. 13.



Curves illustrating the expression $W = \frac{K_1}{\left(1+\frac{1}{x}\right)}$ or $\frac{W}{K_1} = \frac{1}{\left(1+\frac{1}{x}\right)}$, and diagram showing dimensions of air-gap of magnet.

54 per cent. of those actually required. Moreover, leakage and reluctance (mainly this) are responsible for a 46 per cent. wastage.

8. *The Output Criterion of the Magnet.*

To obtain the quantity upon which the output from any given electro- or permanent-magnet depends, it is necessary to deduce the acoustic power in terms of the flux density and the dimensions of the air-gap.

The acoustic power is

$$W = i^2 R_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where i is the coil current and R_a the radiation resistance. For any given type of diaphragm

$$R_a = uC^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where u is a parameter dependent upon the diaphragm and the frequency. The current is governed by the coil impedance and the resistance of the power valve. The former varies throughout the audible register. However, this is immaterial, as it can be shown that a magnet designed for greatest output at a particular frequency gives this condition at any other frequency. At the electro-mechanical resonance frequency—where the coil reactance vanishes—the impedance is purely resistive. For maximum output—not maximum efficiency—R the coil resistance referred to the anode circuit of the valve must be p times the resistance of the latter, *i. e.*, $R = p\rho_v$, where p depends upon the valve characteristics. Taking the most general case where a transformer is used and assuming it to be perfect, the turns ratio $s = [R/R_m]^{1/2}$ where R_m is the total effective resistance of the coil in the secondary circuit including radiation and diaphragm losses. For any given power valve the total resistance in the anode circuit for maximum distortionless output is

$$R + \rho_v = (p+1)\rho_v = \text{a constant.}$$

Moreover, the alternating anode current I corresponding to a definite grid voltage swing is also constant.

Thus the coil current

$$i = Is = I[R/R_m]^{1/2}$$

or

$$i^2 = K_1 R_m^{-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where K_1 is a constant $= I^2R$ the power dissipated in the load.

Now

$$R_m = R_c + R_i + R_d + R_a,$$

where R_c = resistance due to copper of coil;

R_i = resistance due to iron loss;

R_d = resistance due to diaphragm loss;

R_a = resistance due to radiation of sound.

The iron and diaphragm losses introduce unknown

quantities, and for the present they will be omitted so that we can write

$$R_m = R_c + R_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From (1) and (4) the acoustic output

$$W = \frac{K_1 R_a}{(R_c + R_a)},$$

or
$$\frac{W}{K_1} = \frac{1}{1 + \frac{1}{x}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $x = R_a/R_c$.

Expression (5) gives a curve of the form illustrated in fig. 13. When $x=0$, $R_a=0$, $W=0$, and there is no radiation (*in vacuo*), whilst as x increases so also does the radiation. Obviously the output depends upon x, and we shall regard it as the criterion of the system.

Now the d.c. resistance of the coil is

$$R_c = \frac{\rho l}{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the length of wire $l = 2\pi r n$, the cross-section of the wire $A = blf_s/n$, and

$$f_s = \frac{\text{total copper cross-section}}{\text{cross-section of gap}}.$$

Substituting in (6) for l and A, we get

$$R_c = \frac{2\pi r n^2 \rho}{blf_s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Thus from (2) and (7),

$$\left.\begin{aligned} x &= \frac{R_a}{R_c} = \frac{uC^2 blf_s}{2\pi r n^2 \rho} = \frac{u(2\pi r n H)^2 blf_s}{2\pi r n^2 \rho} \\ &= \frac{u}{\rho} . 2\pi r blf_s H^2 \\ &= \frac{u}{\rho} . H^2 V f_s \end{aligned}\right\}, \quad . \quad . \quad . \quad (8)$$

where $V = 2\pi r b l$ is the volume of the annular air-gap.

So far as the magnet is concerned, the criterion is obviously

$$H^2 V f_s, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

this being 8π times the magnetic energy associated with the

metal of the coil. When the air-gap length l is constant, f_s can be assumed constant also, and the criterion reduces to H^2A as shown in a former paper *.

In specifying a magnet, it is obvious that a statement of the flux density by itself has no value, since the gap dimensions are left out of account. Also a statement of H^2V is meaningless, for of two magnets with equal values of H^2V the air-gap of one might be inadequate to accommodate a suitable coil. This is where the space factor becomes important. With a gap of 0·16 cm., f_s has a value of from 0·4 to 0·5, whereas for a 0·08 cm. gap f_s varies from 0·2 to 0·3. Consequently, the energy output from the smaller air-gap is less than that from the larger.

The problem can be viewed from another angle. With an air-gap of 0·16 cm., we could accommodate 50 turns of wire having a resistance R_c, whereas in the more restricted space available with a 0·08 cm. gap, the diameter of the wire would be much smaller and its resistance appreciably in excess of R_c. Thus the dead loss in the latter case would exceed that in the former, with a corresponding reduction in current and, therefore, in sound output.

It follows that the specification of a magnet should be accompanied by the quantity H^2V and the gap dimensions, so that f_s can be computed.

By hypothesis the H^2Vf_s criterion applies when the coil reactance is zero. If the best magnet is selected on the H^2Vf_s basis, it will fulfil the optimum condition in the lower register of an actual reproducer where iron and diaphragm loss occur. The output in the upper register is influenced by the mass, diameter, etc. of the coil, but no definite relationship has been established analytically. Moreover, it is out of the question to incorporate this in the preceding analysis.

A point of interest arises when economical considerations are waived. Assume we have a magnet with $l=0·96$ cm., the remaining quantities being equal to that of another magnet with $l=0·16$ cm. From the above criterion the output with the former magnet would appreciably exceed that from the latter. The ratio of the outputs is not $\frac{0·96}{0·16}\cdot\frac{f_s}{f_s'}$, since $W \propto \frac{1}{1+\frac{1}{x}}$, and it is not proportional to H^2Vf_s since the latter varies as x. With a 0·96 cm. gap a coil of 50 turns can have a very low resistance indeed, but the

* 'Wireless World,' p. 604 (Nov. 26th, 1930).

reactance will be unaltered. Thus the increased output will only be felt over a limited band of frequencies where the copper loss is an important fraction of the impedance. At the same time the large mass of the coil—unless aluminium were used—would restrict the amplitude causing a reduction in the output particularly at the higher frequencies. Moreover, a gap of this length is of no practical value. If H^2Vf_s remains constant, an increase in radius of the coil is accompanied by a larger output. The inductive and motional capacitive reactances both increase proportionately to the square of the radius, so that the increase in output is again limited to a definite frequency band, *unless the current is constant at all frequencies.* Under the latter condition the internal resistance of the valve is so high that there is little damping of the natural oscillations, which assume undue prominence. The preceding argument shows that the quantity H^2Vf_s must be used with discretion.

We are now in a position to deal with the factor $H^2V/8\pi$ which is sometimes cited by manufacturers as a criterion. The magnetic energy stored in the air-gap is $H^2V/8\pi$, but it does not immediately follow that this quantity is a panacea which incorporates all the virtues and vices of a magnet. As we have seen above, the output from a loud speaker is only proportional to H^2V when the radiation resistance is small compared with the copper resistance. This is approximately true for a number of reproducers where the resonances are relatively weak. But it is inapplicable to a moving-coil reproducer of the horn variety where the radiation resistance is 0·3 that of the total resistance. Also, we have shown by preceding examples that H^2V has to be used in conjunction with other information connected with the reproducer. Moreover, it should be clear that $H^2V/8\pi$ does not tell the whole story by any means, and that details of the air-gap are a necessary adjunct.

9. *Rectification Effect of Coil in Non-uniform Magnetic Field.*

Consider the arrangement sketched in fig. 14. The circular coil is immersed in a radial field whose intensity varies linearly from the point O. If A_1 is an extreme position of the coil the alternating current in it is represented by I_1, and the force urging it in the direction of the arrow A_1 is proportional to H_1I_1. If A_2 be the other extreme position of the coil corresponding to current I_2, the force urging it in the direction A_2 is proportional to H_2I_2.

During acceleration in the A_1 direction the mean force is greater than that during the A_2 direction. The reverse is true during the deceleration period. Thus the coil will move further in the A_1 than in the A_2 direction, which is tantamount to saying there is a translational component in the A_1 direction; whence the coil will travel towards the zero field position, and the action can be regarded as one akin to rectification.

Fig. 14.

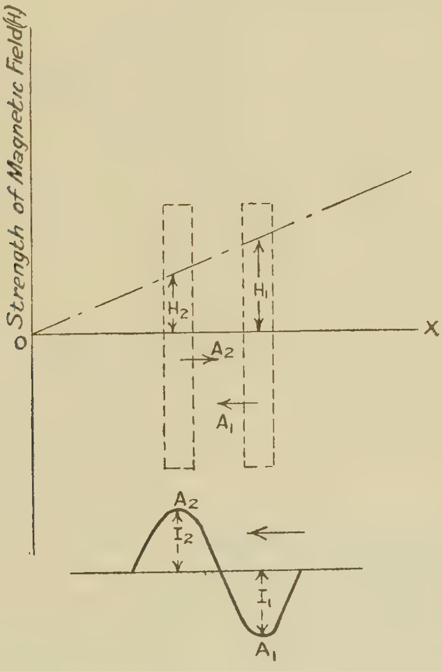


Diagram illustrating coil driven by sine wave current in non-uniform magnetic field.

This phenomenon can be demonstrated in striking fashion by aid of a moving-coil reproducer. If the diaphragm is freely suspended by four threads, the coil when supplied with alternating currrent will move out of the magnet and oscillate about an external point depending upon the gravitational restoring force due to obliquity of the threads. The larger the current the greater the distance the coil is driven from the magnet. A considerable axial force is required to restore the coil to its normal position in the

magnet, and when this force is removed the coil moves out again.

The effect is due to non-uniformity of the field distribution within and without the magnet as shown in previous papers*. The coil does not come out of the magnet until the axial amplitude exceeds a value adequate to bring it into the external leakage field to such an extent that the gravitational control is overcome. Keeping the coil current constant, the frequency must be reduced to a value where this condition is fulfilled. The greater the current the higher the frequency at which translatory motion just ceases to occur. There is obviously a limit due to heating of the coil. Under normal conditions the phenomenon is readily obtained below 100 cycles. If the diaphragm has a powerful radial mode simultaneously and is mounted horizontally, the spectacle is reminiscent of a soaring bird or a helicopter.

In the reproducer used for these tests the coil always moved out of, but not into, the magnet. This is due to the leakage field being less outside than inside the magnet. Doubtless by starting with the coil well inside the magnet it would move inwards.

The analytical expressions for this case are identical with (1) and (2) in Phil. Mag. vii. p. 1014 (1929), excepting that C. the force on the coil per unit current, is now variable or $C=f(x)$. Whatever simplifications are made the equations are unfortunately insoluble for a sine-wave current in the coil.

10. *Impulse Records and their uses.*

In a previous paper we saw that severe impulsing occurred when a rectangular wave form was applied to the grid of a thermionic valve having a reproducer in its anode circuit. The question now arises as to the utility of this test in studying the physical behaviour of diaphragms. Impulse records show the oscillations which are not damped out by the magnetic field or other corresponding influence. With large diaphragm moving-coil apparatus the acoustic damping is small. At low frequencies the magnetic damping is paramount, whilst at high frequencies transmission loss assists the magnetic field. In cases where small diaphragms are associated with long exponential horns, considerable damping is due to the resistive load of the horn, as well as to the magnetic field. Records illustrating the influence of the horn are given in figs. 15 *a* and 15 *b* (Pl. II.).

* Phil. Mag. xi. p. 39 (1931); also 'Wireless World,' *loc. cit.*

Probably the best way of demonstrating the use of impulse records is to give a number of typical examples. In fig. 16 *a* (Pl. III.) we have an example taken from a moving-coil reproducer. It consists of a complex oscillation having low-frequency components of 33 and 200 cycles on which is superposed an oscillation of 2000 cycles. The 33-cycle oscillation is that of the diaphragm as a whole on the surround, whilst the 200 ∼ is due to the surround *per se* acting as an auxiliary resonant diaphragm *. Fig. 16 *b* (Pl. III.) shows a record of the same diaphragm after removal of the rubber surround. The 33 and 200 cycle oscillations have vanished, but the 2000 cycles oscillation due to the main symmetrical mode (2 nodal circles) of the diaphragm persists. A very strong magnetic field would be required to extinguish this mode.

Fig. 17 *a* (Pl. IV.) is a record for a free-edge paper diaphragm, 5×10^{-2} cm. thick, 16·7 cm. radius, apical angle 160°. Although this diaphragm exhibits strong radial modes when steady low-frequency alternating current is used, none is visible on the record. If excited at all by impulsing, such modes are curbed by the magnetic field. For several cycles the oscillation of fig. 17 *a* (Pl. IV.) is a fair approximation to a damped sine wave, but latterly it degenerates into a more complex type. The natural frequency is that of the main symmetrical mode corresponding to two nodal circles.

The above records were taken with the microphone on the axis of the diaphragm. When the microphone is placed at a point remote from the axis the effect is quite different, owing to the focussing or beam effect and to the velocity of propagation of sound in the diaphragm being less than that of sound in air. Fig. 17 *b* (Pl. IV.) is a record for the preceding diaphragm taken with the microphone 25 cm. *away* from the axis. At such a distance, which is approximately equal to the diameter of the diaphragm, interference is considerable, since one side of the diaphragm is much nearer to the microphone than the other. Moreover, the record must be interpreted in this sense.

The set of records in fig. 18 (Pl. V.) illustrates the influence of axial and angular distance upon the wave-form of the transient oscillation. The effect of increase in axial distance is to reduce the oscillations—due to the main symmetrical

* A better example of this complex type of oscillation is given in the 'Wireless World,' p. 15 (May 13, 1931). The ripples on the horizontal portions of some of the records are due to interference from a power system.

mode—which follow the first peak. A standing wave-effect arises, but it is not established until after the peak occurs. There is also diffraction at the microphone. The combination of these two may enhance the output when the microphone is near the diaphragm. The standing wave-effect decreases with increase in axial distance. The influence of angular distance is similar to that displayed in fig. 17*b* (Pl. IV.). The records of fig. 18 (Pl. V.) correspond to a standard diaphragm mounted on a rubber surround which resonated *per se* at 129·5 cycles per second. There is no trace of this on record. Moreover, if the surround resonance occurs below a certain frequency the magnetic field is adequate to render the motion aperiodic. The resonance of the diaphragm vibrating as a whole on the surround at 18·7 cycles is damped out completely. The aperiodic state is seen by the decay of the high-frequency oscillation superposed on an exponential curve. The latter represents the diaphragm being forced back to its equilibrium position by the surround, but restrained from oscillation by the magnetic field. Comparison should be made with fig. 16 *a* (Pl. III.), where both of the surround oscillations occur.

Finally, fig. 19 (Pl. I.) illustrates the case of a diaphragm in which the surround constraint was high enough to promote oscillations at 80 cycles per second. The reproduction from this instrument was accompanied by a strong boom *.

These examples demonstrate the utility of the impulse method in studying the natural oscillations of a diaphragm. Owing to irregularity in some of the records it is difficult to ascertain the frequencies of any but the more powerful oscillations. Broadly speaking, we are able to ascertain the frequency of the main symmetrical mode and to determine whether the mode due to the surround itself or that of the diaphragm on the surround is rendered aperiodic by the magnetic field †. The behaviour of the diaphragm can also be examined by the sudden application of definite frequencies to the grid of the valve, *i. e.*, interrupted sine waves of square modulation. Comparison of results at resonant and non-resonant frequencies would indicate what is to be expected in practice.

The vibrational frequencies of a diaphragm can also be

* A simple method of finding the resonance frequency of the diaphragm on the surround is described in 'Wireless World,' August 8th, 1928. The resonance of the surround *per se* is found by bridge measurement.

† It is interesting to reduce the magnetic field to a value where the motion becomes oscillatory.

found by the inverse of the preceding. The reproducer is set in a highly damped enclosure or in free air and impulsed acoustically. The latter can be accomplished by the impact of two bodies whose natural frequencies are much higher than those of the reproducer—supersonic preferred. The coil current is amplified and recorded in the usual way. Since the response as a microphone falls away with rise in frequency, a correction circuit may be required in the amplifier. If the wave-form is immaterial this circuit is unnecessary.

By associating a light coil or an equivalent electrostatic arrangement with any structure its vibrational frequencies can be found. It does not follow, of course, that the relative magnitudes will be identical with those when the structure is impulsed electromagnetically.

In taking impulse records care must be exercised to ensure that oscillations or aperiodic effects due to transformers or to choke-condenser combinations are not superposed upon those due to the instrument under test. This has been discussed in previous papers dealing with such tests (' Wireless World,' April 3rd, 10th, Aug. 7th, 14th, 1929).

April 1931.

XI. *The Transient Response of the Triode Valve Equivalent Network.* *By* W. Jackson, *M.Sc., A.M.I.E.E., Lecturer in Electrical Engineering, College of Technology, Manchester* *.

THE behaviour of triode valve amplifier systems in the steady state under periodic input voltages has received considerable attention, and Colebrook's generalized analysis † of the triode valve equivalent network permits the deduction of the general character of the variation of the important quantities of the network with the load in the anode circuit. The nature of the transient response of such a network is likely to prove an equally important consideration in design, in view of the increasing use of thermionic valves in telegraphic systems and for such special purposes as increasing the sensitivity of oscillographs, in both of which applications it is desirable that the distorting transients introduced by the elements of the triode circuit shall be of short duration in

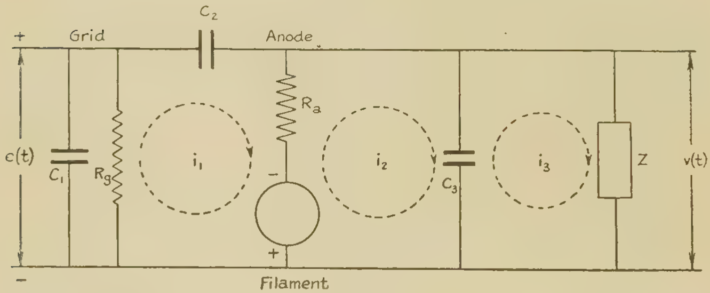
* Communicated by Prof. Miles Walker, M.A., D.Sc.
† Journ. I. E. E. lxvii. p. 157 (1929).

comparison with the duration of any transient voltage pulse to be amplified.

The equivalent network of a single-valve stage as developed by Colebrook is given in fig. 1. The voltage amplification factor and the internal A.C. resistance of the valve are designated μ and R_a ; C_1, C_2, and C_3 are the grid-filament, anode-grid, and anode-filament self-capacities respectively ; R_g the effective resistance of the input circuit and Z the load impedance.

The problem is to derive a general expression for the voltage, $v(t)$, developed across the output or load circuit Z on the sudden application of a voltage, of the form $e(t)$, to the input circuit of the valve.

Fig. 1.



The descriptive differential equations of the network are therefore :

$$i_1\left\{R_a+\frac{1}{pC_2}\right\}-i_2(R_a)=(\mu+1).e(t), \quad . \quad (1)$$

$$i_1(R_a)-i_2\left\{R_a+\frac{1}{pC_3}\right\}+i_3\left(\frac{1}{pC_3}\right)=\mu.e(t), \quad . \quad . \quad (2)$$

$$i_2\left(\frac{1}{pC_3}\right)-i_3\left\{\frac{1}{pC_3}+Z(p)\right\}=0, \quad . \quad . \quad . \quad . \quad (3)$$

where

$$p=\frac{d}{dt}.$$

Eliminating i_1 and i_2 from these equations gives

$$i_3=\frac{(pR_aC_2-\mu)}{R_a+Z(p)\{1+pR_a(C_2+C_3)\}}.e(t). \quad . \quad . \quad (4)$$

The output voltage $v(t)$ developed across the load $Z(p)$ is therefore given in terms of the circuit parameters by

$$v(t) = Z(p) \, . \, i_3 = \frac{(pR_aC_2 - \mu)Z(p)}{R_a + Z(p)\{1 + pR_a(C_2 + C_3)\}} \, . \, e(t). \quad (5)$$

If the voltage $e(t)$ be applied to the system at the time $t=0$, it may be expressed in the notation of the Heaviside Operational Calculus by $e(t).[1]$, where $[1]$ is the unit functional voltage of value zero for time $t < 0$ and unity for time $t > 0$.

The complete expression for the output voltage $v(t)$, steady and transient, is then given as the solution of the operational equation

$$v(t) = \frac{(pR_aC_2 - \mu)Z(p)}{R_a + Z(p)\{1 + pR_a(C_2 + C_3)\}} \, . \, e(t) \, . \, [1]. \quad . \quad (6)$$

The form of the operating impedance function $Z(p)$ for the load will depend on the nature of the circuit employed for coupling the triode-valve stage under consideration to a measuring device or to succeeding stages of amplification. As this circuit may contain all of the parameters resistance, inductance, and capacity, the expression for $Z(p)$ will, in general, be complex in nature. Further complexity in the solution of equation (6) results from the fact that it is not always possible to utilize the whole of the voltage developed across $Z(p)$, for application to a succeeding stage, because of the need for providing isolation of the grid of this stage from the high-tension supply.

The most convenient method of obtaining the solution of equation (6) depends upon the form of the applied voltage $e(t)$. If a convenient equivalent operator is available for $e(t)$, as is the case for periodic and exponential time variations, direct solution by the "Expansion Theorem"* may give easiest solution.

If, however, the desire is to consider the factors affecting the nature of the transient solution, this may be effected by determining the response of the system to the unit voltage [1] as the solution of the operational equation

$$h(t) = \frac{(pR_aC_2 - \mu)Z(p)}{R_a + Z(p)\{1 + pR_a(C_2 + C_3\}} \, . \, [1]. \quad . \quad . \quad (7)$$

The complete response to the functional voltage $e(t)$ is then conveniently obtained by substituting the value of $h(t)$ obtained as the solution of equation (7) in the "Superposition Theorem."

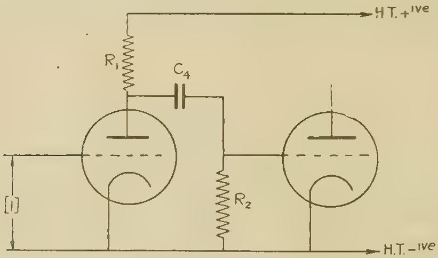
* See Appendix.

Nature of the Transient Response in typical Amplifier Networks and the Effect of the Self-capacities.

Resistance-capacity Coupling.

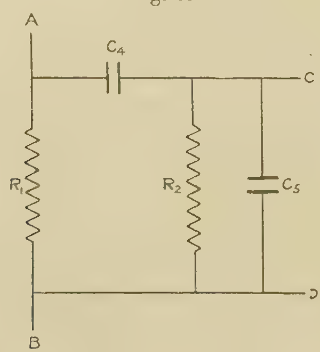
The resistance-capacity coupled amplifier, accepted as best for the distortionless amplification of complex periodic audio-frequency voltages, is shown diagrammatically in fig. 2, and its equivalent load circuit is drawn in fig. 3.

Fig. 2.



C_5 represents the input capacity of a succeeding triode stage, while the input resistance of this stage is included in the leak resistance R_2.

Fig. 3.



The operational equation for the voltage developed across the load terminals AB is given by equation (7), where $Z(p)$ is the generating impedance of the entire circuit between A and B, and is given by

$$Z(p)=\frac{R_1\{1+pR_2(C_4+C_5)\}}{1+p\{R_2(C_4+C_5)+R_1C_4\}+p^2(R_1R_2C_4C_5)}.$$

The voltage applied to the next stage between C and D is, however, only

$$\frac{pR_2C_4}{\{1+pR_2(C_4+C_5)\}}$$

of this voltage. The output voltage developed by the system, on the application of unit voltage [1] to the input, is therefore given by the solution of the operational equation

$$h(t)=\frac{pR_1R_2C_4\{R_aC_2p-\mu\}}{(R_1+R_a)+pC_4\{R_1R_2+R_2R_a+R_aR_1\}+p^2C_4R_1R_2R_a(C_2+C_3+C_5)}[1]. \quad (8)$$

In obtaining this equation, whenever C_2, C_3, C_5 have occurred added to C_4 they have, justifiably, been neglected as small in comparison.

Solution of this equation by the "Expansion Theorem" shows that the output voltage $h\,(t)$ is made up of two transient components of time constants

$$\frac{1}{a\pm\sqrt{a^2-b}},$$

where

$$a=\frac{R_1R_2+R_2R_a+R_aR_1}{2R_1R_2R_a(C_2+C_3+C_5)}$$

and

$$b=\frac{R_1+R_a}{R_1R_2R_aC_4(C_2+C_3+C_5)}.$$

For normal values of the circuit constants, b is small compared with a^2, and the time constants reduce to $\frac{1}{2a}$ and $\frac{2a}{b}$; that is,

$$T_1=\frac{R_1R_2R_a(C_2+C_3+C_5)}{R_1R_2+R_2R_a+R_aR_1}$$

and

$$T_2=\frac{C_4(R_1R_2+R_2R_a+R_aR_1)}{R_1+R_a}.$$

For circuit constants of the following typical values:

$\mu=20$; $R_a=2\times10^4$, $R_1=10^5$, $R_2=10^6$ ohms;

$C_2=C_3=5\times10^{-12}$, $C_5=10\times10^{-12}$, $C_4=5\times10^{-10}$ farads,

giving a steady state amplification to periodic voltages of approximately 16·4, the solution of equation (8) is

$$h(t)=16{\cdot}4e^{-3{\cdot}04\times10^6t}-16{\cdot}4e^{-1968t}$$

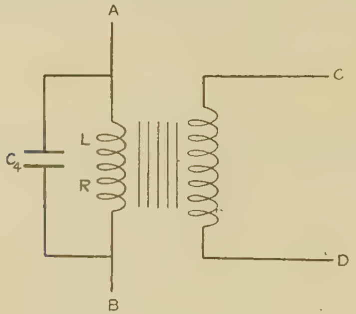
When the stray capacities C_2, C_3, C_5 are made equal to zero, T_1 becomes zero. Solution of the simplified operational equation resulting from this approximation gives

$$h(t) = 16{\cdot}4e^{-1968t}.$$

It is apparent therefore that, apart from producing zero instantaneous response to the input voltage by providing instantaneously a short-circuit path through the network, the effect of the stray capacities—provided they are of normal values—can justifiably be neglected in deriving the transient response.

Further, that for given values of R_a, R_1, and R_2, the time constant of the local transient may be reduced by reduction

Fig. 4.



in the value of the coupling condenser C_4, to which the time constant T_2 is directly proportional.

Transformer Coupling.

A simple equivalent load circuit of the transformer coupled low-frequency amplifier is shown in fig. 4, in which C_4 represents the equivalent capacity of the transformer primary and secondary windings and of the input circuit of the succeeding stage transferred to the primary. This total capacity can now be included in C_3, with which it is in parallel, given a load impedance $Z(p)$ equal to $(R+Lp)$.

From equation (4) the current i_3 through this load is the solution of

$$i_3 = \frac{(pR_aC_2 - \mu)}{R_a + (R+Lp)\{1+pR_a(C_2+C_3)\}}\left[1\right],$$

and therefore the output voltage between C and D is given by the solution of the operational equation

$$h(t)=\frac{(p\mathrm{R}_a\mathrm{C}_2-\mu)\mathrm{M}p}{\mathrm{R}_a+(\mathrm{R}+\mathrm{L}p)\{1+p\mathrm{R}_a(\mathrm{C}_2+\mathrm{C}_3)\}}[1] \quad . \quad (9)$$

for the unit input voltage.

The solution of this equation shows $h(t)$ to consist of two component transients of time constants

$$\frac{1}{a\pm\sqrt{a^2-b}},$$

where

$$a=\frac{\mathrm{L}+\mathrm{R}\mathrm{R}_a(\mathrm{C}_2+\mathrm{C}_3)}{2\mathrm{R}_a\mathrm{L}(\mathrm{C}_2+\mathrm{C}_3)} \quad \text{and} \quad b=\frac{\mathrm{R}+\mathrm{R}_a}{\mathrm{R}_a\mathrm{L}(\mathrm{C}_2+\mathrm{C}_3)}.$$

When, however, all self-capacities are assumed equal to zero, there appears only one transient component of time constant $\frac{\mathrm{L}}{\mathrm{R}+\mathrm{R}_a}$. Taking the same valve as before and associating with it a typical transformer for which $\mathrm{L}=100$ henrys, $\mathrm{R}=2000$ ohms, $\frac{\mathrm{M}}{\mathrm{L}}=3{\cdot}5$, and C_3 (including any condenser which may be permanently connected, inside the transformer casing, across the transformer primary winding) of value 3×10^{-10} farads, the solution of equation (9) becomes

$$h(t)=70e^{-1{\cdot}66\times10^{5}t}-70e^{-220t}.$$

With C_2 and C_3 taken as zero, the simplified operational equation provides a solution

$$h(t)=-70e^{-220t},$$

so that the same conclusions as before can be drawn concerning the importance of stray capacities in affecting the form of the transient response.

It is at once apparent that the local disturbing transient occurring in a transformer coupled amplifier, made up of components such as are typical of those employed for the amplification of periodic voltages, is much more prolonged than that present in a resistance-capacity coupled amplifier.

Choke Coupling.

For this type of coupling the equivalent load circuit may be drawn as in fig. 5. Treating this circuit in the same

manner as with the resistance-capacity coupling, the operational equation for the output voltage between C and D is

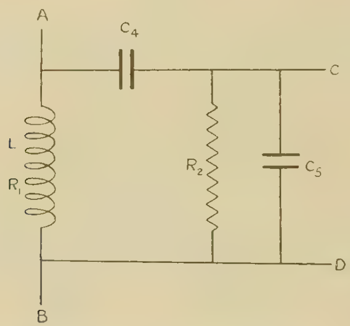
$$h(t)=\frac{R_2C_4p(R_1+Lp)\{pR_aC_2-\mu\}}{(R_1+R_a)+p\{L+C_4(\Sigma R_1R_2)\}+p^2\{C_4[L(R_2+R_a)+R_1R_2R_a(\Sigma C)]\}+p^3\{R_2R_aLC_4(\Sigma C)\}}\cdot[1],$$

where

$$(\Sigma C)=C_2+C_3+C_5 \quad\text{and}\quad (\Sigma R_1R_2)=R_1R_2+R_2R_a+R_aR_1.$$

Since the denominator is a cubic, the output voltage $h(t)$ must consist of three component transients. Solution in the general case is, however, troublesome, and the nature of the response in the simple case of $C_2=C_3=C_5=0$ will

Fig. 5.



give the important information. The above operational equation then reduces to

$$h(t)=\frac{-\mu R_2C_4p(R_1+Lp)}{(R_1+R_a)+p\{L+C_4(\Sigma R_1R_2)\}+p^2C_4\{L(R_2+R_a)\}}\cdot[1],$$

the solution of which consists of two transient terms of time constants

$$\frac{1}{a\pm\sqrt{a^2-b}},$$

where

$$a=\frac{L+C_4(\Sigma R_1R_2)}{2C_4L(R_2+R_a)} \quad\text{and}\quad b=\frac{R_1+R_a}{C_4L(R_2+R_a)}.$$

Taking the circuit constants used for the resistance-capacity coupled case, with the modification that the resistance R_1 is now replaced by a choke of inductance L 100 henrys and resistance R_1 2000 ohms, the complete

expression for $h(t)$ in response to the unit input voltage given by

$$h(t)=2{\cdot}26e^{-220t}-21{\cdot}86e^{-1960t}.$$

This circuit gives a steady state amplification to 50 cycle per second input voltages of the same value as that obtained from the previously considered resistance-coupled amplifier. The first term, in spite of its smaller amplitude, is the more disturbing component because of its slower decay with time. Its time constant $\frac{1}{220}$ corresponds very approximately to that of the circuit composed simply of the choke and the internal resistance of the valve $\frac{L}{R_1+R_a}$.

The effect on the form of this output voltage of giving different values to C_4, the intervalve coupling condenser, has been calculated. For a value of C_4 equal to 10×10^{-10} farads—*i.e.*, twice as large as before—$h(t)$ is given by

$$h(t)=5{\cdot}20e^{-220t}-24{\cdot}80e^{-975t},$$

while for a value of 1×10^{-10} farads

$$h(t)=0{\cdot}37e^{-220t}-19{\cdot}97e^{-9780t}.$$

Decreasing C_4, therefore, produces two desirable effects; it reduces the amplitude of the more troublesome transient and rapidly reduces the time constant of the other. It must be borne in mind, however, that as C_4 is decreased the voltage step up of the stage for periodic input voltages is decreased, particularly at the lower audio frequencies, because of the increasing voltage lost across C_4.

With both the transformer and choke coupled amplifier, indeterminate transient components are liable to be present in the output voltage if the inductance in the anode circuit varies with the current passing through it. This danger does not arise in parallel-fed circuits.

Parallel-fed Choke Coupling.

The equivalent load circuit of a parallel-fed choke coupled amplifier is shown in fig. 6, where the self-capacity of the choke L and of the input of the succeeding stage are represented by C_5. Proceeding as before, and taking $C_2=C_3=C_5=0$, the output response of the circuit to the unit input voltage is given as the solution of the operational equation

$$h(t)=\frac{-\mu R_1C_4p(R_3+Lp)}{(R_1+R_a)+pC_4(\Sigma R_1R_2)+p^2C_4L(R_1+R_a)}[1].$$

The solution of this equation shows $h(t)$ to be composed of two transient components of time constants

$$\frac{1}{a \pm \sqrt{a^2-b}},$$

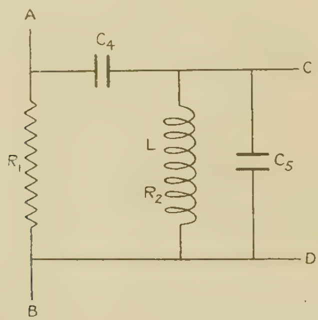
where

$$a=\frac{\Sigma R_1R_2}{2L(R_1+R_2)} \quad \text{and} \quad b=\frac{1}{C_4L}.$$

Taking the following component values as typical of those found in circuits employed for periodic voltage amplification, $\mu=20$; $R_a=2\times10^4$, $R_1=10^5$, $R_2=2000$ ohms; $L=100$ henrys, and $C_4=5\times10^{-8}$ farads, the complete expression for $h(t)$ is given by

$$h(t)=-\{16{\cdot}66\cos 2\pi.70.t-2{\cdot}98\sin 2\pi.70.t\}e^{-88{\cdot}3t}.$$

Fig. 6.



With the large values of C_4 employed to avoid low note loss in amplifiers designed for audio-frequency amplification, it is seen that the transient response is likely to be periodic and dies away exponentially at a comparatively slow rate.

The critical value of C_4 for non-periodic transient response occurs when $a^2=b$, that is, when

$$C_4=4L\left\{\frac{R_1+R_a}{\Sigma R_1R_2}\right\}^2,$$

ΣR_1R_2, as before, being $(R_1R_2+R_2R_a+R_aR_1)$.

Appendix.

The operational equation given in (7) may be written in the form

$$h(t)=\frac{M(p)}{D(p)}.[1], \quad \ldots \ldots \quad (10)$$

where $M(p)$ and $D(p)$ are certain functions of p. In the cases discussed in the paper $M(p)$ is never of higher power in p than $D(p)$, so that direct solution of this equation by the "Expansion Theorem" results in the following manner.

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the n roots of $D(p)=0$. Provided that none of these roots are zero and that there are no equal roots, the complete solution of (10) is given by

$$h(t)=\frac{M(0)}{D(0)}+\sum_{k=1}^{k=n}\frac{M(\lambda_k)}{\lambda_k \cdot D'(\lambda_k)} \cdot e^{\lambda_k t}$$

where $M(0)$ and $D(0)$ are the values of $M(p)$ and $D(p)$ for $p=0$, and $D'(\lambda_k)$ is the value of $\frac{dD(p)}{dp}$ for $p=\lambda_k$.

If, having obtained the response of the network to the unit voltage $[1]$, it is desired to derive the response $v(t)$ to a functional input voltage $e(t)[1]$, this may be obtained by substituting for $h(t)$ and $e(t)$ in the "Superposition Theorem"

$$v(t)=e(t) \cdot h(0)+\int_0^t E(t-\phi) \cdot h'(\phi) \cdot d\phi$$

$$=e(t) \cdot h(0)+\int_0^t E(\phi) \cdot h'(t-\phi) \cdot d\phi,$$

where ϕ is a variable of integration and

$$h'(\phi)=\left[\frac{dh(t)}{dt}\right]_{t=\phi}.$$

If, in equation (10), $M(p)$ is of higher power in p than $D(p)$, $\frac{M(p)}{D(p)}$ can always be reduced to the sum of a number of terms, some of which are integral powers of p and one of which is of the form $\frac{x(p)}{y(p)}$, where $x(p)$ is not of higher power in p than $y(p)$. The resultant expression may then be

$$Ap[1]+B[1]+C\frac{x(p)}{y(p)}[1],$$

when the solution of equation (10) is the sum of the solutions of

$$Ap[1], \; B[1], \text{ and } C\frac{x(p)}{y(p)}[1].$$

XII. *Photography of Diffraction Patterns due to small Circular Apertures.* *By* L. R. WILBERFORCE, *M.A., Lyon Jones Professor of Physics, The University of Liverpool* *.

[Plate VI.]

THE diffraction effects produced when light from a point-source passes through a circular aperture in a thin plate and is then received on a screen have been very fully investigated by Lommel †, and his paper has been frequently referred to by English writers, but the mathematical form in which his results were exhibited has left their details unfamiliar to a not inconsiderable number of physicists, and it may be of service to describe their main features in more popular language.

The changes in the diffraction pattern due to variations in the distances of the aperture from the source and from the screen, in the diameter of the aperture, and in the wave-length of the light employed, may all be summed up in two statements.

(1) The character of the pattern depends only upon the number of half-period elements (which we may abbreviate as h–p) which the aperture transmits; (2) the linear dimension or "scale" of the pattern is proportional to the distance apart on the screen of the interference fringes which would be due to two imaginary identical point-sources of light at opposite ends of a diameter of the aperture. We may call this distance a "fringe-unit."

The character of the pattern may be described under two subheadings: (a) the positions of the maxima and minima, (b) the distribution of intensity at and between these positions.

Lommel's results show that if the aperture does not transmit more than about $1h$–p the character of the pattern is absolutely constant, so far as the positions of the minima are concerned, while the positions of the maxima and the distribution of intensity tend asymptotically to constancy as the size of the aperture is decreased. Further, the radii of successive minima, starting from the centre of the picture, increase by distances which soon become practically equal and whose value is then one fringe-unit.

* Communicated by the Author.

† *Abh. d. K. Bayer Akad. d. Wissensch.* xv. 1884–1886.

This statement may be made more definite numerically. Lommel's results are expressed in terms of a quantity Z, such that its relation to the distance ζ of a point from the centre of the pattern and the length f of a "fringe-unit" may readily be proved to be given by $Z=\pi\zeta/f$. The positions of the minima are given by the roots of the equation $J_1(Z)=0$. It can readily be calculated from a table of Bessel's Functions that the corresponding values of ζ/f are: 1·298, 2·223, 3·238, 4·241, 5·243, 6·244, 7·245, 8·246, 9·246, 10·246, 11·247, 12·247, etc.

When the aperture transmits more than 1 h–p the character of the pattern, as is well known, becomes more variable. It is then necessary to consider separately the portion of the screen in the geometrical shadow and the portion which would be fully illuminated if light travelled in straight lines, which we may call the geometrical disk. The phenomena within the disk have often been described and photographed and are so familiar that it is unnecessary to refer to them further, but it is less generally realized that in the geometrical shadow the light does not steadily decrease to zero but does so through a series of steps of minima and maxima, and that the successive values of ζ/f for the minima of these steps continue to be given by the series of numbers already written down, beginning, however, not at its first term but at such a term that the value of ζ calculated from it will correspond to the point nearest the edge of the geometrical shadow.

The fact that at points well within the geometrical shadow the character of the fringes will tend to constancy may be explained as follows. Let a plane wave pass normally through a circular aperture of radius r and let the light be received on a screen at a distance b from the aperture large compared with r, and let us consider the amplitude at a point P on the screen whose distance ζ from the axis is small compared with b but large compared with r. If N is the projection of P on the plane of the aperture, the loci of points in the aperture equidistant from P will be circular arcs with N as centre, and as the distance of N from the aperture is large compared with r it will be a good approximation to replace these arcs by parallel straight lines.

If y and $y+dy$ are the distances of two such lines from the centre of the aperture, the distance from P of the strip which they enclose is

$$\sqrt{b^2+\zeta^2} \pm \frac{y\zeta}{\sqrt{b^2+\zeta^2}}$$

and the area of the strip is

$$2dy\sqrt{r^2-y^2}.$$

Thus, by the usual formula, if λ is the wave-length and A the amplitude of the light at the aperture, the vibration at P is

$$\frac{2A}{\lambda}\int_{-r}^{+r} dy \frac{\sqrt{r^2-y^2}}{\sqrt{b^2+\zeta^2}} \sin\frac{2\pi}{\lambda}\left(vt-\sqrt{b^2+\zeta^2}+\frac{y\zeta}{\sqrt{b^2+\zeta^2}}\right).$$

We may write b for $\sqrt{b^2+\zeta^2}$ and, expanding the trigonometrical term and noticing that the coefficient of $\cos\frac{2\pi}{\lambda}(vt-b)$ vanishes between the limits of integration, we reduce the expression to

$$\frac{2A}{\lambda b}\sin\frac{2\pi}{\lambda}(vt-b)\int_{-r}^{+r} dy\sqrt{r^2-y^2}\cos\frac{2\pi\zeta y}{\lambda b}.$$

If we write x for y/r, the amplitude of this vibration will be

$$\frac{2r^2A}{\lambda b}\int_{-r}^{+r} dx\sqrt{1-x^2}\cos\frac{2\pi\zeta r}{\lambda b}x.$$

This is
$$\frac{2r^2A}{\lambda b}\cdot\frac{\pi}{Z}\cdot J_1(Z),$$

where Z is written for

$$\frac{2\pi\zeta r}{\lambda b}.$$

If the aperture transmits m half-period elements

$$m\lambda = r^2/b,$$

and if the "fringe-unit" is f,

$$f = \lambda b/2r,$$

and the expression for the amplitude becomes

$$2m\pi A J_1(\pi\zeta/f)\Big/\frac{\pi\zeta}{f}.$$

That is, in the region of the pattern where ζ is large compared with r, the character of the pattern involves the number of half-period elements merely as a factor governing the absolute intensity of illumination, while the relative

intensities at corresponding points and the positions of the maxima and minima will be the same for all values of m.

So far as I have been able to ascertain, no adequate photographs of the diffraction patterns when m is not greater than unity have been published. Arkadiew *, who gives a fine series of pictures for larger apertures and for opaque disks, has photographs for $m=\frac{1}{2}$ and $m=1$ which show nothing outside the first considerable minimum. This is explicable by the fact that the intensity within this minimum is so great in comparison with that of the rest of the field that an exposure sufficiently long to show the outer portions of the pattern will lead to such an over-exposure of the centre that the plate will be spoilt. I therefore tried to remedy this by interposing a suitable mask in front of the photographic plate, so that a sufficiently long exposure could be given to the marginal part of the picture, and afterwards, by removing the mask, a short exposure to show the central portion. Though I naturally had expected that it would ultimately be necessary to use monochromatic light, preliminary experiments were undertaken using as a source of light a small approximately circular hole fixed a few centimetres in front of the crater of an arc. A magnified "pin-hole picture" of the crater was thus formed in the plane of the aperture, and was adjusted so that the aperture was completely within this picture. In this way a maximum of illumination was conveniently obtained. The mask, which consisted of a brass disk of suitable small diameter cemented on a thin plate of good glass, was carried by a mechanical stage, and thus could be accurately adjusted horizontally and vertically so as to be central with the pattern while observed through a suitable eye-piece.

Although the experiments were made in a darkened room the requisite exposures were often so long that further precautions against stray light became necessary. A blackened wood box about a metre long and of 20 cm. square section was made, having a sufficient opening at one end and an arrangement for holding the photographic plate at the other. The stage carrying the mask was attached to a heavy solid rod passing into the box near the bottom and close to the photographic plate attachment through a short tube which it did not touch. The mask could thus be made central with the pattern before the plate was put into position, and any slight displacement of the box due to the subsequent insertion of the plate would not disturb the adjustment of

* *Phys. Zeit.* xiv. p. 832 (1913).

the mask. When the marginal portion of the plate had been sufficiently exposed the mask could be lowered without disturbance of the plate by rotating the rod through a right angle about its axis, and the supplementary exposure thus obtained.

The photographic plates used were backed Wellington Ortho Process plates, and a preliminary negative obtained with an aperture transmitting about $\frac{1}{4}$ h–p revealed an unexpected but fortunate coincidence. A long series of maxima and minima was shown—in fact, eight or nine maxima could be measured and four or five more faintly seen, and their final disappearance even then seemed to be due to evanescent intensity rather than to any blurring due to overlapping. It appeared, therefore, that these plates were abnormally sensitive to a comparatively narrow band in the spectrum of the crater, and all the advantages of approximately monochromatic light were obtained without the serious decrease of illumination inseparable from the conventional methods.

Measurements of the diameters of some minima were made and the mean wave-length of the operative band of the spectrum thus determined, with the following results.

Aperture (1).—Diameter ·086 cm., distance from plate 335 cm., radius of 9th minimum 1·425 cm.

From the values of ζ/f for the minima previously given it follows that

$$f = 1{\cdot}425/9{\cdot}246 = {\cdot}154 \text{ cm.}$$

Hence $\lambda = {\cdot}086 \times {\cdot}154/335 = 3{\cdot}96 \times 10^{-5}$ cm.

Aperture (2).—Diameter ·0775 cm., distance from plate 335 cm., radius of 8th minimum 1·40 cm.

Thus $f = 1{\cdot}40/8{\cdot}246 = {\cdot}170$ cm.

Hence $\lambda = {\cdot}0775 \times {\cdot}170/335 = 3{\cdot}93 \times 10^{-5}$ cm.

Aperture (3).—Diameter ·065 cm., distance from plate 325 cm., radius of 10th minimum 2·025 cm.

Thus $f = 2{\cdot}025/10{\cdot}246 = {\cdot}198$ cm.

Hence $\lambda = {\cdot}065 \times {\cdot}198/325 = 3{\cdot}95 \times 10^{-5}$ cm.

Aperture (4).—Diameter ·044 cm., distance from plate 335 cm., radius of 5th minimum 1·55 cm.

Thus $f = 1{\cdot}55/5{\cdot}243 = {\cdot}296$ cm.

Hence $\lambda = {\cdot}044 \times {\cdot}296/335 = 3{\cdot}88 \times 10^{-5}$ cm.

These values for λ, which correspond to the extreme of the visible violet end of the spectrum, are in remarkable agreement, remembering that the diameters of the minima can only be measured to the nearest half millimetre.

Figs. 1, 2, & 3 (Pl. VI.) give prints from the negatives furnished by apertures (4), (3), and (1). The distance from the aperture to the source was in each case equal to its distance from the photographic plate. Considerable difficulty was experienced in making suitable apertures of the smaller diameters. The diffraction patterns were at first found to display great unsymmetrical variations of intensity even when the circular form of the aperture was beyond doubt. These irregularities were finally traced to the burr which every method of drilling which could be devised left behind it. Even the process of depositing a thick layer of copper electrolytically on each side of a piece of platinum foil, drilling completely through with a watch maker's drill, and then dissolving off the copper with nitric acid did not completely eliminate it. The technique which finally proved successful was to use platinum foil about ·08 mm. thick, which was of sufficient mechanical strength not to buckle, to work up the hole to nearly the required diameter by fine watchmakers' broaches turned rapidly in a small lathe, and to keep the burr on each side rubbed down flat on a fine Arkansas stone with water. This, of course, left the edges of the hole ragged and possibly slightly out of truth. The foil was then softened by heating to bright redness in a Bunsen burner, and, after cooling, each side of the hole was in turn just pressed squarely on to the tapering portion of a needle of slightly larger diameter than the hole while this needle was being rapidly rotated in a small lathe. This left the hole smooth and circular without producing a new burr.

The same amount of care was fortunately unnecessary for the small holes which were fixed in front of the arc to serve as approximate point-sources. It was sufficient to secure that in each case the shape should be roughly circular and, in order to avoid blurring, that the diameter should not exceed one-half of a fringe-unit multiplied by the ratio of the distance of the aperture from the source and screen respectively.

The use of an opaque mask and the consequent sharp line of demarcation produced on the negative spoils the aspect of the diffraction pattern in its immediate neighbourhood. In order to study this aspect in the neighbourhood of the centre for apertures transmitting nearly 1 *h–p* some masks

of a graduated opacity were produced photographically, and figs. 4, 5, & 6 (Pl. VI.) are prints from the negatives obtained, corresponding to $\frac{1}{2}$ *h–p*, 1 *h–p*, and $1\frac{1}{2}$ *h–p* respectively. The same aperture was used for each at the same distance (335 cm.) from the plate, and the size of a fringe-unit is thus the same throughout ; the distances of the source were 335 cm., 112 cm., and 67 cm. respectively. The slight variations of the patterns for $\frac{1}{2}$ *h–p* and 1 *h–p* are fairly well brought out. The mask used for fig. 6 (Pl. VI.) transmitted too much light in its marginal portion, and the earlier minima are not well shown in consequence.

Figs. 7, 8, 9, & 10 (Pl. VI.) are the results of using a larger mask and long exposures so as to exhibit the maxima and minima far removed from the centre. They correspond to $\frac{1}{2}$ *h–p*, 1 *h–p*, and $1\frac{1}{2}$ *h–p*, obtained as explained above, and 2 *h–p* obtained by reducing the distance of the source to 48 cm., and, as before, the fringe-unit is the same for each. The coincidence in radii of the outer maxima and minima for the various pictures is shown in fig. 11 (Pl. VI.), where half of the picture for 2 *h–p*, with its familar dark centre, and half of that for $\frac{1}{2}$ *h–p* are put together.

The exposures for fig. 10 (Pl. VI.) were 60 minutes for the margin and 30 seconds for the centre, which indicate the rate of attenuation of the intensity ; the exposures for figs. 7, 8, & 9 (Pl. VI.) had similar ratios.

For fear of misunderstanding it should be explained that the negatives obtained diminished so greatly in density from the centre outwards that their details could not be transferred to a print by a single exposure, and two exposures, using suitable graduated masks, were needed. Thus the pictures given, while they show the *positions* of the maxima and minima, do not reproduce the relative intensities at different portions of the field.

I must not conclude without expressing my great indebtedness to my lecture-assistant, Mr. F. J. Welch, for the skill and patience which he has lavished on the development and printing of these photographs.

George Holt Physics Laboratory,
The University of Liverpool.
July 27, 1931.

XIII. *On Sensitive Flames.* *By* G. B. Brown, *M.Sc.*, *Lecturer in Physics, University College, London* *.

[Plates VII.-X.]

Introduction.

SINCE its first accidental discovery by John Leconte in 1858 † the phenomenon of sensitive flames has received the attention of a large number of investigators. In spite of this, however, a detailed explanation of the phenomenon has never been given, and the object of the present investigation is to examine the nature of the instability of gaseous jets more closely and to attempt to discover the causes underlying their sensitivity to the vibrations of sound.

The first thorough examination of sensitive jets was made by Tyndall at the suggestion of Leconte ‡. Tyndall used a number of different kinds of jets, but the most sensitive ones were made by drawing out ordinary glass tubing. Using tuning-forks he found that frequencies of 3200, 2400, 2000, and 1600, in this order of effectiveness, reacted upon his most sensitive flames. Tyndall also showed that flames such as the batswing and the fishtail, originally insensitive to sound, could be made to jump visibly when a whistle was sounded, if a current of air from a slit was brought against the flame so as to make it flutter. Even a candle proved sensitive when distorted by air from a blowpipe. Later, by passing the gas over ammonia and then hydrochloric acid, so as to produce a smoke of sal ammoniac, Tyndall showed that similar effects are obtained with unlit gases. He used unlit coal gas, hydrogen, carbon dioxide, and air. In all cases the pitch effective was found to be much lower than in the case of flames.

Tyndall concluded that the effect of sound on the jets of gas was to produce a condition of turbulence similar to that caused by increasing the pressure: "We bring it to the verge of falling, and the sonorous pulses precipitate what was already imminent. This is the simple philosophy of all these sensitive flames."

Barrett, Tyndall's assistant at the Royal Institution, had a different theory as to the cause of sensitivity in flames, and

* Communicated by Prof. E. N. da C. Andrade, D.Sc., Ph.D.

† "On the Influence of Musical Sounds on the Flame of a Jet of Coal Gas," Phil. Mag. 4th ser. xv. p. 235 (1858).

‡ Phil. Mag. 4th ser. xxxiii. pp. 92 & 375 (1867).

in the 'Philosophical Magazine' for 1867 * he ventures to put it forward. He quotes Tyndall's explanation that "an external sound added to that of a gas-jet already on the point of roaring is equivalent to an augmentation of pressure on the issuing stream of gas". He then points out that the flames flare with the slightest increase in *velocity*. Since the sound waves must also throw the tubing through which the gas flows into vibration, "the flow of gas is thereby driven from the sides and urged more towards the centre of the tube, and the current, thus confined within narrower limits, must issue from the burner with an increased velocity so long as the sound continues. It is the greater rapidity thus induced in the issuing stream of gas which causes the flame to shorten and diverge . . . ."

He supports this curious hypothesis with various observations, but it does not seem to have occurred to him to test it by shielding first the tubing and then the flame itself. I find that if this is done with an efficient screen no difference is observed in the former case, while in the latter the sensitivity disappears almost entirely.

The next investigator to take up this question was Lord Rayleigh. He examined the behaviour of a sensitive flame when placed in a region of stationary waves, and observed that it was disturbed at the antinodes and not at the nodes, and consequently was affected at points where the ear would not be, and *vice versa* †.

Later Rayleigh ‡ examined phosphorus smoke-jets by intermittent vision, and noticed that the sound waves produced a "serpentine motion of the jet previous to rupture." He made use of resonators in order to increase the sensitiveness of the jets to the frequency of the tuning-fork used, which was 256. Rayleigh had difficulty in getting sufficient illumination for his stroboscopic experiments, and he repeated his observation with liquid jets. He also examined fishtail burners with liquid and found that they were sensitive, and "when much excited, throw out tall streamers in the perpendicular plane." He concludes, however, that even with the best arrangement as to sensitiveness and intermittent vision, the appearances presented by the liquid jets are difficult to interpret and also to reproduce in a drawing. Rayleigh used a "water-engine" to provide uniform rotation for his stroboscopic disk, and this no doubt prevented his examining

* Phil. Mag. pp. 219 & 287 (1867).
† 'Scientific Papers,' i. p. 406.
‡ *Loc. cit.* ii. p. 268.

gas flames, which require a much greater frequency than liquid jets in order to show sensitivity. He assumed, however, that the phenomena occurring in the gas flame when disturbed by sound were similar to those observed with liquid jets issuing into liquid and with smoke jets, *i. e.*, the column becomes sinuous, and when the amplitude has reached a certain value disrupts, causing "flaring," and in special cases forking. Rayleigh also wrote several mathematical papers on the stability of jets, which will be referred to later.

Present Investigation.

The investigations described below were undertaken at the suggestion of Professor E. N. da C. Andrade, and are a continuation of some research on jets originally commenced by him.

It was hoped by means of different frequencies produced by a valve oscillator, in conjunction with an amplifier and moving-coil loud-speaker, to discover the cause of the sensitivity of gas flames, and to examine, in particular, whether resonance in the jet and tubing had any effect in determining the frequencies to which the flames would most readily respond.

It has been found that the variation of sensitivity with frequency is very complex and that the range of frequencies to which different jets respond varies considerably. Nearly every jet examined has shown a marked directional effect when the direction of the sound is perpendicular to the axis, and the most sensitive ones are those in which this effect is most pronounced. In these cases the column of gas forks when in the most disturbed position, and becomes unaffected when the jet is turned about its axis through a right angle. The nature of the instability induced by sound in smoke jets has been examined stroboscopically and by means of photography. It has been found to consist of an undulatory motion of the column of smoke previous to forking, which very closely resembled that which is observed in the case of a jet of gas which is impinging on a wedge and producing edge-tones. It is suggested that there may be a close connexion between the two phenomena.

In some very rare cases flames can be made to fork without the aid of sound by introducing some form of obstruction into the jet, such as wire or cotton-wool. The flame then gives out a nearly pure note of its own, but it has not been found possible to repeat these results at will.

Experimental Method.

Source of Sound.

A heterodyne oscillator in conjunction with an amplifier and Rice-Kellogg moving-coil loud-speaker was used as the source of sound. Since it was of the first importance that the wave form given should be pure and as free as possible from harmonics, and that the amplitude should remain constant, an oscillator based on one designed by H. Kirke and described by him in the 'Wireless World'* was constructed. It can easily be shown that if two valve oscillators are heterodyning, and if one current is stronger than the other, the resultant rectified current is proportional to the weaker of the two (assuming a linear rectifier). In other words, with a linear detector the amplitude of the rectified beats is unaltered by a change in the amplitude of the stronger oscillation. Further, harmonics can only occur in the resultant rectified current if both the high frequency oscillators have harmonics.

Consequently in this heterodyne set the weaker oscillator contains filter circuits to remove harmonics, and very weak coupling, and its frequency and amplitude are kept constant. The frequency of the stronger oscillation can be varied by means of condensers, so that when heterodyning with the weaker oscillation, frequencies from 0 to 10,000 can be obtained. If the above conditions obtain these frequencies should be free from harmonics and, moreover, of constant amplitude.

Resistance-capacity amplification is used with low resistances consisting of tapped potentiometers of 50,000 ohms. Two of these had ratios of 1 : 2, and the third a ratio of 1 : 1·05 between each tapping. A very large range of known fractions of the total output could thus be obtained by varying the tappings on the potentiometers. Care was taken that the potentiometers were non-reactively wound. This was done by winding them on a bobbin with a large number of sections, alternate sections having the winding reversed. The variable oscillator had three condensers: the first was used as a zero-adjusting condenser by setting it so that the beat frequency as indicated by the milliammeter in the detector anode circuit was zero; the second was then used to increase the frequency from 0 to 3000, and the third from 3000 to 10,000. A ·3 μF. air-condenser was also included in the circuit when frequencies up to 17,500 (the

* Vol. iv. no. 41, p. 67 (1927).

limit of response of the Rice-Kellogg speaker) were required.

The Rice-Kellogg speaker was fixed in the centre of a square baffle-board of 7-ply wood 3 ft. × 3 ft. This was placed on a table which also supported the oscillators and amplifier, care being taken that the oscillators, which were carefully shielded with copper-sheet, were separated by at least 5 ft. to avoid any residual interaction. Facing this table was another of the same height which supported the clamps for holding the jets, and which also had attached to it rails on which a movable board, identical with the one to which the loud-speaker was attached, could run backwards and forwards. This board, the purpose of which is explained shortly, had attached to it a small brass frame with a very fine wire stretched across it, and this moved over the surface of a wooden metre scale screwed to the table. By this means the position of the moving baffle-board could be read on the scale. At the back of this table was a large heavy felt curtain which stretched from the ceiling to the floor (15 ft.) and extended in a horizontal direction for 10 ft. This reduced the effect of reflexion to a negligible amount. (Even with the board at the distance of the curtain, the effect on the flames was very small.)

Measurement of the Frequency.

The simplest way of calibrating the condenser readings of the variable oscillator was obviously to make use of the well-known fact that sensitive flames readily indicate the positions of the antinodes when placed in a region of stationary waves. The reflecting board was placed on its rails and moved backwards, readings being taken of successive positions for maximum disturbance of the flame which was placed between it and the loud-speaker. In this manner by taking means the frequency could be obtained with 1 per cent. accuracy over the range for which the flames were sensitive. A check was obtained at lower frequencies by observing beats between tuning-forks of known frequency and the oscillator.

Apparatus for Supply of Gas and Air.

Since most jets require a pressure above that of the ordinary gas supply, some form of pressure-pump had to be employed. It was found that one of the type manufactured by Edwards and Son answered very well for this purpose. These pumps will run smoothly at speeds as low as 1 or 2

revolutions per sec., and can be used to increase the supply-gas pressure of, say, 6 cm. of water by as little as half a centimetre. The impulses of the pump were smoothed out by allowing it to discharge into a large tank to which the tubing supplying the jet was connected through a tap. In the actual apparatus used the pump was arranged so that by means of four taps it could be rapidly changed from pumping gas into one tank, to pumping air into a separate container. A water manometer was connected to the outlet from both tanks. It was always found better to regulate the pressure at the jet by altering the rheostat in series with the pump motor, rather than by means of taps or screw clips. The reason for this was the fact, first observed by Tyndall, that in some cases if the pressure on a sensitive flame is increased and the flame brought back to the condition of just not flaring by means of a tap, the flame is no longer sensitive. He came to the conclusion that an essential condition of sensitivity was "that a free way should be open for the transmission of the vibrations from the flame, *backwards*, through the gas-pipe which feeds it. The orifices of the stopcocks near the flame ought to be as wide as possible" *.

Rayleigh † showed that the insensitivity produced by the stopcocks was due to the fact that flaring was prematurely produced. There are two ways in which this may be caused. Either the irregular flow through the tap may cause ricochetting of the current of gas from side to side, as Barrett suggested ‡, or sound to which the jet is sensitive may actually be produced at the tap, and the tubing act as a speaking-tube in conducting these vibrations to the orifice.

Rayleigh introduced various nozzles into the supply tube in order to deflect the stream, but he found no tendency to flare unless a hissing sound could be heard. He therefore favoured the second explanation.

To test Lord Rayleigh's theory, a high-pitched whistle to which the jet used was very sensitive, was placed in a wide-bore glass tube in series with the rubber tubing supplying the jet. It was just possible to hear when the whistle was sounding by placing the ear near to the outside of the wide-bore tube. It was invariably found that when the whistle sounded the flame became disturbed and its sensitivity much impaired.

* Phil. Mag. 4th ser. xxxiii. p. 92 (1867).

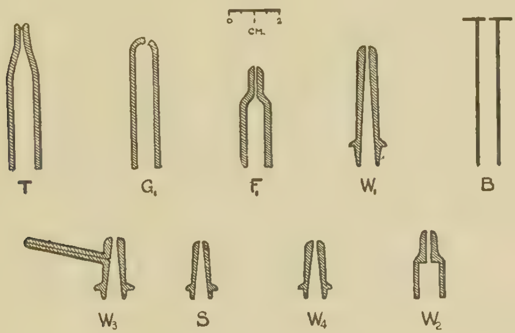
† Scientific Papers,' ii. p. 101; also Phil. Mag. xiii. pp. 340–347 (1882).

‡ *Loc. cit.*

This kind of disturbance is the drawback in using gas cylinders for the supply, since with these it is very difficult to avoid some sort of hissing noise at the valve or the pressure regulator. This difficulty can, however, be eliminated by packing the tubing near the jet with cotton-wool.

It was found that the diameter and the length of the metal or rubber tubing used for connecting the reservoir to the jet had no effect on the behaviour of the flame. The position of the tubing near the jet, however, makes a very great difference, since it alters the manner in which the gas streams out of the orifice. This should always be adjusted by trial for the maximum sensitivity, or for best forking, or whatever behaviour is most desired.

Fig. 1.



Types of Jets used.

As has been mentioned the original observation of sensitivity to sound was made by Leconte on a "fishtail" gas-burner. Tyndall, on taking up the investigation, used drawn-out glass tubing, often with a V-shaped cut in the orifice * (fig. 1, T). Rayleigh used high-pressure pin-hole steatite gas-burner jets of the type invented by Mr. Sugg, and also drawn out glass jets.

Another kind of sensitive glass jet for low pressures has been described by Sutherland †. A glass tube of about 1 cm. diameter is rotated with one end in a blowpipe flame until the end very nearly seals itself up. While still soft it is slightly flattened, so as to make the orifice noticeably elliptical (fig. 1, G). The sensitivity is greatest in the direction of the major axis of the ellipse. "This shape

* W. F. Barrett, Phil. Mag. 4th ser. xxxiii. p. 219 (1867).

† G. A. Sutherland, 'Nature,' cviii. p. 533 (1921).

provides the sudden change of pressure on which sensitiveness depends. . . . As with high-pressure jets, the flame loses sensitiveness if the orifice be too nearly circular, or if its ellipticity be too great." This latter statement is not in agreement with the findings of Jordan and McClung *, who made special search for the most delicate detector of faint sounds.

According to them the orifice must be accurately circular for greatest sensitivity, and they recommend a metal jet 2 in. long, tapering from 0·113 in. diameter at the entrance down to ·043 in. at the exit. The inside must be perfectly smooth. These dimensions were the result of experiments with many different shapes and sizes of jets. In the present research a large number of jets of all kinds were examined: these are classified under several headings below.

1. *High-pressure Jets.*

(a) *Ordinary Glass Tubing.*

Straight pieces of ordinary glass tubing of diameter up to about half a centimetre were found to show slight sensitivity. This did not consist of ducking, but of movements of the visible yellow portion in the upper part of the flame.

Pressures from 5 cm. to 32 cm. of water.
Range of frequencies 850–6300.

Independent of whether the orifice was rough or annealed.

(b) *Capillary Tubing.*

Glass capillary tubes 20 cm. long and 1·3 mm. diameter were tried. These showed very slight sensitivity to the rattle of keys, but if two or more tubes were bound together so that their orifices were in one plane the sensitivity was increased. Tubes of 5 cm. and 10 cm. length were also examined with the same result.

(c) *Drawn-out Glass Jets* (*Tyndall type*) (fig. 1, T).

A large number of jets of the type used by Tyndall were examined. These had a V-shaped slit filed in the orifice, and this was then annealed to prevent cracking. Glass tubing of diameter about 1 cm. was drawn out to diameters of between 1 and 2 mm.

* Jordan and McClung, Proc. Roy. Soc. Canada, xviii. p. 197 (1924).

The sensitivity and performance varied considerably. Some flames increased their height when disturbed by sound; others showed very marked forking; others showed a very prononnced directional effect, *i. e.*, when in their most disturbed state due to sound, if they were turned through 90°, they resumed their undisturbed appearance.

The range of sensitivity varied from 500–9000 in the case of the best, down to 3000–6000 for the worst. The most effective frequency varied, being either 3300, 4600, or 5850. Some were sensitive to keys but not to any noise made by the speaker, *i. e.*, sensitive above 17,500.

(d) *Round-ended Glass Tubing* (*Sutherland type*).

Some of these were very sensitive. The most sensitive jet used was one of this type (fig. 1, G_1). This jet forked perfectly and had very marked directional properties. If held at the entrance of the cone of the loud-speaker a turn of 90° would cause it to drop from an undisturbed flame of 40 cm. to a forked flame 7 cm. high at the fork and 12 cm. at the tips. Another turn of 90° and it became undisturbed again. It was sensitive for frequencies between 500 to 10,000. The diameter was approximately 0·13 cm.

Other jets varied in sensitivity and range, several being insensitive in the range of the loud-speaker although sensitive to keys. In some cases an unsymmetrical flame was produced, and this showed sensitivity only in one part.

G_1 was still sensitive when the amplitude was reduced to 1/2000th of the maximum output, *i. e.*, intensity reduced to 1/4,000,000th, which was scarcely audible.

(e) *Brass Jets.*

These were of two kinds: (1) Cylindrical and conical jets of various forms designed by Professor Andrade, and (2) brass tubing closed at the top by a sheet of metal in which holes of various shapes and diameters were cut.

The jets of the first kind are shown in fig. 1, nos. W_1 to W_4. These were designed specially to see whether resonance in the orifice of the jet was a factor in determining the sensitivity:

W_1. The orifice was distinctly elliptical. Major axis ·120 cm., minor axis 0·114 cm. Most sensitive

when the minor axis was parallel to the direction of sound.

Pressure ≏ 16·4 cm. water.
Range 3250–18,000.
Most effective frequency 5850.
Slight forking.

W_2. Pressure ≏ 8 cm. Diameter 0·160 cm.
Range 2300–11,700.
Most effective frequency 5850.
Very sensitive.

W_3. Pressure ≏ 7 cm. Diameter 0·219 cm.
Range 900–about 10,000.
Most effective frequency 4600.

This jet was supplied with a tube leading into the main jet through a very small hole. No difference in sensitivity was found to occur when this was placed facing the loud-speaker or in any other position.

W_4. Pressure ≏ 36 cm. Diameter 0·112 cm.
Sensitive to keys, but not within the limits of the loud-speaker (0–17,500).

The jets of the second kind (fig. 1, B) were made by soldering a thin sheet of brass across the top of a brass tube 6 cm. long and diameter 0 85 cm. In the sheet were drilled circular holes of different diameters, and in some, line slits, square holes, triangular holes, etc. were punched.

The jets with circular holes were made of different sizes in a series from nos. 52–57 (0·161 cm. to 0·110 cm.). These holes, when first made, were not polished at the edges, and it was found very difficult to get the flames to burn steadily ; some produced notes which went up and down the scale, caused originally by draught disturbance. In this state it was impossible to get some of the jets to show any sensitivity at all.

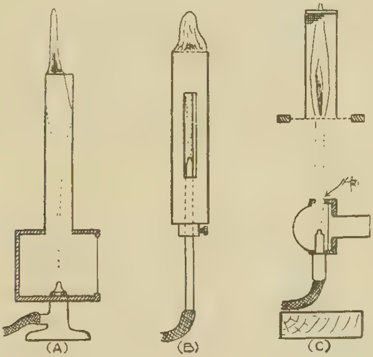
When the holes were rounded off, however, by lightly spinning a countersinking drill in them, the sensitivity appeared. The sensitivity and range varied markedly from one to the other, and no simple relation held between the pressures and the diameters of the holes or the starting frequencies, except that on the whole there was a tendency for the larger diameters to have the lower pressures and lower frequency starting-points.

This bears out the finding of Roberts*. By means of a special method of boring the holes in a sheet of metal he succeeded in proving that the pressure for maximum sensitivity and also for flaring decreased as the diameter of the hole increased. But even with the greatest care in boring the hole he never succeeded in getting the flaring pressure the same on both sides of the hole. It is evident that microscopic irregularities in the construction of the jet affect the behaviour of a stream of gas passing through it very greatly.

(f) *Sugg's Steatite Burners.*

Some of Sugg's Pinhole Steatite burners, S_1, S_2, and S_3, were examined (fig. 1, S).

Fig. 2.



One of them (S_2) was found to be very sensitive, while another (S_3) did not respond to any noise made by the speaker, but was very sensitive to keys. The orifices of these steatite burners are extremely irregular although the insides are fairly smooth. The diameters were 0·091 cm., 0·112 cm., and 0·069 cm. respectively.

2. *Low-pressure Jets.*

(a) *Rayleigh's Flame.*

This well-known type of flame, which is shown diagramatically in fig. 2 (A), was examined with regard to frequency and range. These flames can be made very sensitive, the flame disappearing down the tube and only returning after several seconds. They are very sensitive to draughts, and loud sounds very

* J. H. T. Roberts, Phil. Mag. xxiii. p. 368 (1912).

easily put the flame out altogether. The actual burner used was one marketed by Messrs. Griffin and Tatlock. It was found to be sensitive to the frequencies

250, 400–430, 1200–1260.

If one of these pure frequencies is maintained the flame executes a cycle of changes—disappearing into the tube, reappearing, becoming unsteady, and then disappearing again, and so on, with a period of several seconds.

(b) *Bunsen Burner.*

The ordinary Bunsen burner is obviously very much the same in construction as the Rayleigh flame, and sensitivity can sometimes be obtained, as in the Rayleigh flame, by turning down the gas pressure until the luminous flame burns lopsidedly. It can be made into a sensitive flame for low notes by attaching a diaphragm to the base of a conical sound receiver fitted to one of the air-inlet holes, any other inlet holes being tightly closed. This has been done by Mache *.

(c) *Gauze Type.*

Jordan and McClung † also describe the construction of a low pressure low-frequency flame, shown in fig. 2 (C). This is similar in principle to Rayleigh's flame, except that the unlit column of gas, instead of passing through the sound-chamber into a tube, passes out through a hole, *p*, of the same diameter as the column, into the air, and is then lit above a gauze which is held some little distance above the top of the chamber.

A somewhat similar type has been designed by Zahradnicek ‡, fig. 2 (B). The gas passes out of a jet into a space surrounded by two concentric brass cylinders 10 cm. long and about 2 cm. in diameter. A slit in each, 5 cm. × 0·8 cm., can be varied in size by rotating one brass tube on the other, which it closely fits. At the top of the tubes is a wire grid (200 holes/cm.^{2}). The gas is lit above the gauze, and the length of the unlit portion may be varied by sliding the jet in and out of the brass tubes. For low notes, according to the

* H. Mache, *Phys. Zeit.* xx. p. 467 (1919).
† *Op. cit.*
‡ *Phys. Zeit.* xxx. p. 555 (1929).

author, this length should be about 5 cm., and for high notes about 2 cm.

A jet of this type was constructed and examined for range of sensitivity. With a distance of $7\frac{1}{2}$ cm. between the jet and the gauze the range was from 1200 to 5000; when this was shortened to $2\frac{1}{2}$ cm. the range did not commence before 3000. The sensitivity was, however, very slight, and the pressure must be adjusted very carefully. The actual range of sensitivity of the jet alone with the gas lit at the orifice was from 3850–7500.

Characteristic Behaviour of Sensitive Flames.

The behaviour of sensitive flames in general is well known. A jet of some kind with an orifice of about 1 mm. diameter is supplied with coal gas. If the pressure is increased the flame first of all burns steadily and increases in height, but a point is reached when an unsteady flickering appearance occurs, and it is in this condition that the flame exhibits sensitivity to sound. Any further increase in pressure causes the height of the flame to decrease suddenly, turbulent motion sets in, and the flame appears markedly disturbed and produces a characteristic roaring noise or "flaring." A state of turbulence can be induced by the action of sound when the flame is just on the point of flaring, and this is the cause of sensitivity. Turbulence sets in at some point in the column of the flame, and this causes the height to decrease and so exhibit the familiar "ducking" of sensitive flames. Fig. 3 (Pl. VII.) shows a flame just on the point of flaring. (Jet W_2) fig. 4 (Pl. VII.) shows the effect of shaking a bunch of keys. The height decreases from 24 cm. to 12 cm. Fig. 5 (Pl. VII.) shows the turbulent state produced by excess pressure without sound.

If instead of a noise composed of many different frequencies, a pure note is maintained, it becomes evident at once that nearly all flames show a directional effect; as the jet is turned round the height of the flame exhibits maxima and minima, and these occur alternately along two directions which are at right angles. Further, if the positions of minimum height are examined it is found that the flame is spread out in a plane containing the axis of the jet and a line joining the jet and the loud-speaker, and the position at which this spreading occurs approaches the orifice as the amplitude increases.

This is particularly noticeable in the case of sheets of flame such as are produced by jets consisting of a straight

slit when the slit is perpendicular to the direction of propagation of the sound. Fig. 6 (Pl. VII.) shows a case of this kind. In some cases spreading may take place in two parallel planes, the flame thus behaving like two very close flames, but usually this spreading occurs in one central plane only.

Fig. 7 (Pl. VII.) shows a case of double spreading, viewed at a slight angle to the planes in which the spreading occurs; the two sheets can be clearly distinguished towards the bottom of the flame.

The height above the orifice at which spreading occurs varies with frequency in a peculiar manner to be described later. In jets which produce a sheet of flame which is not symmetrical, being higher on one side than the other (fig. 24, Pl. X.), sensitivity may occur for different frequencies in different parts. One such jet examined was sensitive from frequencies of 700 to 1700 on one side and from 2700 to over 10,000 on the other. In special cases this spreading takes the form of a fork, and the issuing jet of flame divides into two separate jets the angle between, which increases with the amplitude of the sound (fig. 8, Pl. VII.).

In the case of flames from line jets the marked spreading or forking of the flame in the direction of propagation of the sound appears to cause the remaining parts of the flame to draw inward and coalesce, and this often causes the height of the flame to *increase* slightly when the sound occurs. The case represented in fig. 7 (Pl. VII.) shows this phenomenon, the spreading being nearly normal to the paper; the sideways contraction of the flame a short distance above the orifice which lead to the increase in height is very marked.

In some very rare cases when the jet was packed with cotton-wool and the flame was roaring, due to excess pressure, it gave out a fairly pure note. If now the frequency of the oscillator was increased gradually from zero, a very high note could be heard in the flame, whose pitch came down gradually to zero and then increased again as the frequency of the oscillator continued to rise. This note became very weak as its frequency increased and gradually became inaudible, exhibiting the characteristics of a beat note. If, however, the pitch of the oscillator was raised still further, another beat note occurred which passed through a minimum in the same manner as the first. This phenomenon was first observed with hydrogen, but was later obtained with coal-gas. In the latter case it was seen that the flame remained steadily in a fork, giving out of itself a nearly pure note of frequency

6300. If now the oscillator was started with this same frequency the fork remained undisturbed ; but if the frequency of the oscillator was either increased or decreased the arms of the fork began to close and open again in time with the beats to be expected. This of course became indistinguishable by the time that the beats became an audible note. Another vibration of the flame which gave beat notes with the oscillator occurred at a frequency of 4300, although in this case the fork was not clearly defined. A somewhat similar observation appears to have been made with a stream of ether vapour by A. T. Jones*. In general when a flame produced a nearly pure note due to excess pressure the stroboscope showed that symmetrical "bulges" starting at the jet were travelling up the flame with a frequency equal to that of the note given out. These effects, however, were very difficult to obtain and could not be repeated with any certainty.

These symmetrical swellings travelling up the flames occur with any pressure if hydrogen is used, and produce a characteristic flickering appearance. This helps to mask the effect produced by sound on the jet, and so hydrogen jets do not show great sensitivity. The swellings are always observed with coal-gas flames from jets whose diameter is greater than about half a centimetre. The well-known flickering of any form of the Bunsen flame can be shown by the stroboscope to consist of such disturbances whose wave-length seems to remain fairly constant at about 4·5 cm. and whose frequency is 3–4 a second.

This kind of disturbance can be caused in coal-gas or hydrogen streams when the orifice is less than half a centimetre by the vibrations produced by the method of the familiar singing-flame experiment in which the flame burns inside an open tube of certain length. It is only necessary to examine such flames stroboscopically to prove that this is what is occurring when the flame "sings."

Although a sensitive flame *which is disturbed owing to the effect of sound* produces usually only a characteristic roaring noise, a note of the same pitch as that of the oscillator can be obtained if a piece of wire or other sharp obstacle is placed in the flame. The exact position is of importance and has to be found by trial In flames which fork the position extends from a point in the centre of the flame 1 cm. below the fork to points about ½ cm. up each of the branches. The effect is most marked when the direction of the wire is at

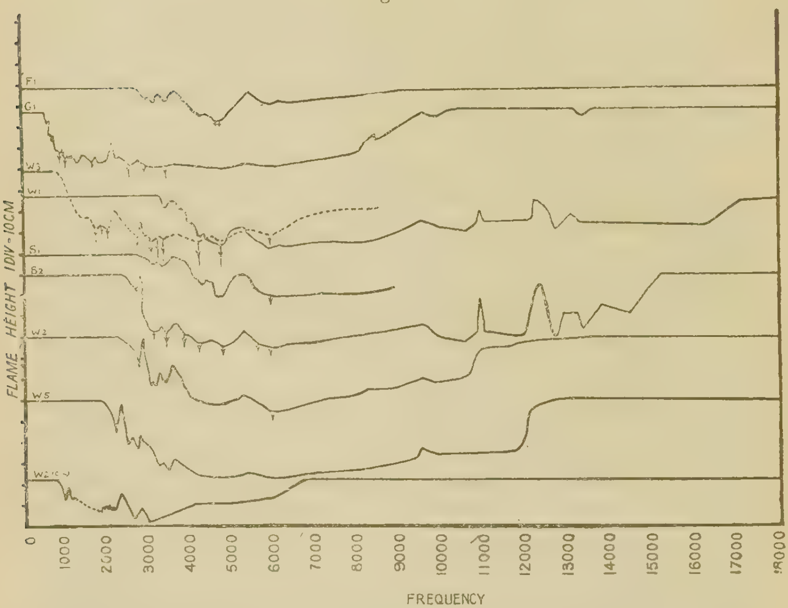
* Thesis for Degree, Clark University, Worcester, Massachusetts, 1916.

right angles to the vibration of the sound. The diameter of the wire makes very little difference.

The Effect of Frequency on the Sensitivity of Jets.

The degree of sensitivity of the flame was measured by observing the height of the flame on a centimetre scale placed near it and to one side, care being taken that its presence did not affect the behaviour of the flame by reflexion from its surface. It was found that as the oscillator was caused

Fig. 9.



to vary its frequency from zero to about 17,500 (the point at which the loud-speaker ceased to respond) the flames remained steady at first, then commenced to duck, and passed through a number of maxima and minima before becoming unresponsive again at the high frequencies.

The height of the flame was plotted against the frequency on a graph (fig. 9). Points on the graph were observed for minima and maxima, and a sufficient number in the intervals between, to show the behaviour of the flame. In the lower range of frequencies there were so many maxima and minima that these were not all measured, and only those which were most marked are shown in fig. 9. The vertical lines indicate

forking, their lengths measuring the depth of the junction of the fork below the top of the flame. Curve W_5 is for a jet exactly similar to W_2 but with the length of the narrow portion doubled (*cf.* fig. 1). The lowest curve is for jet W_2 packed up to the orifice with cotton-wool.

The results of this investigation showed :—

(1) That the different jets varied very much in regard to the range over which they were sensitive. Some begin early in the range and finish early ; some begin late and finish late. The poor jets show their lack of sensitivity by beginning late and ending early and by having much less marked maxima and minima than good jets.

(2) That provided a number of jets are all sensitive in a given range, *they all show maxima and minima at the same frequencies*, regardless of the shape of jet, the size of jet, the pressure, and the nature of the connecting tubing. Any alteration in these latter merely alters the range of the sensitivity, the sharpness of the maxima and minima, and their relative size. The most marked minima occurred at frequencies of approximately 5850, 4600, 3300, and 2400.

(3) That frequencies above those produceable by the Rice-Kellogg speaker are effective in causing disturbance in the flames. Jets were found which gave no response at all in the range 0–17,500, but which answered readily to the shaking of keys and of coins. Also some jets which were sensitive in the loud-speaker range seemed to be much more sensitive to keys, and would duck much lower when keys were sounded as well as the loud-speaker. The jet consisting of a straight slit actually *increased* its height when keys were rattled near it (see p. 174), and this was particularly noticeable when the oscillator was sounding, since for the frequencies so produced it decreased its height, as is the usual case.

It was, of course, of great importance to show that the form of the flame-height against frequency curves was a property of coal-gas jets, and not due to other causes, such as

(*a*) presence of harmonics produced by the oscillators and amplifier ;

(*b*) nature of the loud-speaker ;

(*c*) reflexion from distant parts of room, surrounding apparatus, and so on.

As regards (*a*), the behaviour of the flames was examined when the plate voltage of the oscillators and the plate voltage

and grid-bias of the amplifier were altered. No difference was observable. Duddell oscillograph records were also taken of the output of the amplifier, and the wave form proved to be a very pure sine wave up to the limit of the oscillograph (about 2000 ∼). Further, it is obvious that in the case of, say, the glass jet G_1, which shows many maxima and minima between 500 and 1000, the effect cannot in any case be due to harmonics lower than the 30th, since such harmonics can all be produced directly by the oscillator, and no response occurs in many cases.

To test the possibility (*b*), the Rice-Kellogg speaker was replaced by a small Stirling loud-speaker of ordinary diaphragm type, and the curve obtained showed that the behaviour of the flame was almost identical.

Finally, with regard to (*c*), the whole experiment was repeated in a different laboratory with the conditions entirely changed. The gas was supplied by a cylinder, the loud-speaker was a Brown type, the oscillator was a single LS. 5-valve, and the frequencies were measured by a bridge method. The jet used was W_3. The frequency of its maxima and minima were found to be exactly as before up to 4500, which was the limit with this particular oscillator.

It seems therefore evident that the fact that all the different kinds of jets used showed the same flame-height against frequency curves expresses a property of gaseous jets, and not one due to causes inherent in the apparatus used.

Jets of hydrogen from a cylinder were also examined. Although these flames cannot be made to show anything like the same degree of sensitiveness as coal-gas flames, nevertheless there is a small degree of "ducking," the frequencies of the minima being those most effective for the coal-gas flames, *i. e.*, 3300, 4600, 5850.

Variation of Response with Amplitude.

By means of the potentiometer tapping in the amplifier it was possible to decrease, by known fractions of the total output, the amplitude of the sound emitted by the loud-speaker. The effect of this was as would be expected : the minor fluctuation in height disappeared gradually and left only the most marked maxima and minima, viz., 2400, 3300, 4600, 5850, which also disappeared when the amplitude was decreased sufficiently. The range of sensitivity also decreased. Fig. 10 shows this effect for jet W_3, the pressure being constant at 6·7 cm. of water. When the intensity was reduced to approximately 1/1000th the jet remained undisturbed.

The range decreased from 1200–8900 to 2200–6300, when the amplitude was reduced to 1/16th.

Fig. 10.

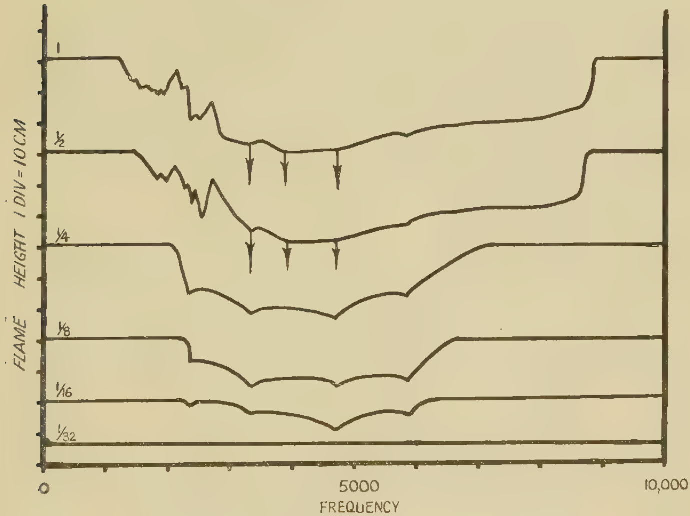
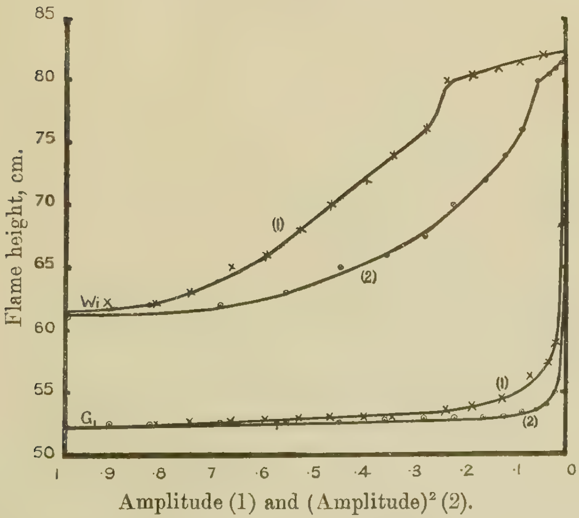


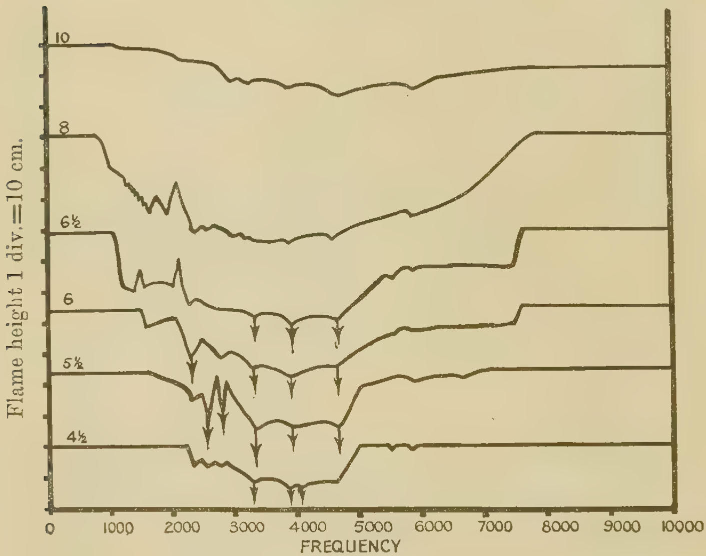
Fig. 11.



If one jet is taken and the height plotted against the amplitude for one given frequency curves such as are shown in fig. 11 are obtained. This shows the results for two jets,

W_1 and G_1, with the frequency adjusted to that for maximum sensitivity, *i. e.*, 4600 ~. The curves marked (2) are those plotted against the amplitude squared. It will be seen on examination that there is an initial period where there is little or no response, then a period where the drop in height is very nearly proportional to the amplitude, and then a period where the height decreases very slowly, tending towards a stationary value. The initial insensitive period for the glass jet G_1 does not show on the scale on which fig. 11, is plotted.

Fig. 12.



Variation of Response with Pressure.

As is well known, sensitive jets show their greatest sensitiveness when just on the point of "flaring". If the pressure is now decreased or increased the response falls off; this is well illustrated by fig. 12, which shows flame height-frequency curves for jet W_3 for different pressures in cm. of water from 10 cm. to $4\frac{1}{2}$ cm. A pressure of 8 cm. is the one for greatest sensitiveness. If the pressure is slightly in excess (10 cm.) the response is not so marked, but the range of sensitiveness extends to higher frequencies. As the pressure is reduced below 8 cm. the minor maxima and minima disappear first, leaving finally the frequencies 2400, 3300,

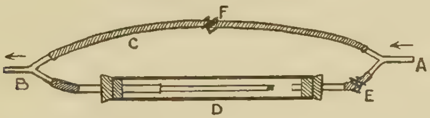
4600. The frequency 5850 has almost disappeared on the curve, since the range of continuous sensitivity has decreased from (800–7600) to (2200–5000). The relative importance of some maxima and minima seems to change with the pressure, *e. g.*, the minima at 2800 and the maxima on each side of it are much enhanced at a pressure of $5\frac{1}{2}$ cm. They hardly appear at all at other pressures.

Stroboscopic Examination of Smoke-jets.

In order to throw more light on the nature of the disturbance produced by sound, experiments were made with air-jets containing cigarette smoke to render them visible. The method of producing them was as follows:—Air from the large tank (pumped by the motor) was led through an arrangement of tubes shown in fig. 13.

This consisted of two glass T-pieces, A and B, connected by the rubber tubing C to act as a by-pass, and by the glass

Fig. 13.



tube D, which was about a foot long and 1 inch in diameter. Its ends were closed by rubber bungs, through which tubes were passed, one being a brass tube which acted as a cigarette holder. The cigarette was first lit and then rapidly placed in the tube D, and air allowed to flow through by opening the screw clip E, the clip F being closed. By this means a stream of air containing fine particles of smoke was obtained. The quantity of smoke could be adjusted by means of the clips E and F. Since cigarettes of ordinary length did not last sufficiently long for close examination of the jet, some cigarettes a foot long were obtained specially. It was found that the cigarettes should not be closely packed in order to give the best results, and since the density of packing varied very much in the sample tried it was important to select the cigarettes carefully first.

The jets of smoke were very unstable, and great care had to be taken to keep the space surrounding them free from draughts. The pressure was found to be much lower (1 or 2 cm. of water), and they were sensitive to notes of much lower pitch than gas-flames.

The behaviour of the smoke-jets was examined stroboscopically. The stroboscope consisted of a brass disk radius 20 cm., with ten holes of diameter $\frac{1}{2}$ cm. The ratio of diameter of hole to opaque portion between was 1 to 7. The disk was attached to the axle of an electric motor capable of rotating at 2000 revs./min. without undue vibration. Light from a 2000 c.p. arc was focussed by condensers on a hole in the disk. Another similar hole was made in a brass plate which was held by a clamp, so that the hole was in line with the beam and close to the rotating disk. The beam was further focussed on the flame by a cylindrical lens. When the frequency of interruption of the light was that of the sound the motions of the jet could be seen with great distinctness either by reflected light or, better still, by observing in a direction opposite to that of the light and a little to one side, so as to avoid the direct glare of the lantern. There are, of course, many speeds of the motor at which the jet appears to be stationary, but vision is always clearest when the frequency of interruption coincides with that of the sound. If there is any doubt that this is the case the stroboscope can be used as a siren by blowing with a glass tube through the holes in the disk. The note heard should coincide with that of the speaker.

Photographs of the smoke-stream were taken first of all by placing the camera facing the beam from the lantern and obtaining the shadow on the plate. The smoke, however, was rarely dense enough to make this a good method, and so finally photographs were taken with scattered light by placing the camera facing the lantern but just out of the line of the main beam. Exposures of half a second were required for the stroboscopic photographs, the plates used being Ilford's Golden Iso-Zenith (speed H & D 1400).

The effect of sound on jets of cigarette smoke is shown in figs. 14 and 15 (Pl. VIII.). At a certain height above the orifice the column of smoke appears to vibrate from side to side in a direction parallel, or nearly parallel, to that of the sound, and this wave-like motion travels up the column and increases in amplitude. When the amplitude of these waves reaches a certain amount the smoke rises in a confused conical mass, the apex facing downwards. In jets, however, which show marked forking the mechanism of the process can be seen quite clearly. As the column undulates from side to side portions of approximately half a wave-length in length are broken off, and then pass on in a straight line alternately up each arm of the fork, first one to the left and then one to the right. This can be seen very clearly in fig. 15

(Pl. VIII.). If the successive portions do not break off sharply a thin filament of smoke, extending from the top of one portion in one arm of the fork to the bottom of the portion above it in the other arm, travels up the space between the arms, causing it to have a horizontally striated appearance.

As in the case of gas-flames, the pressure must be near that of turbulent motion. If the pressure is too small the waves either do not increase in amplitude or may even decrease and die out (fig. 16*b*, Pl. VIII.). If the pressure is increased the forking becomes a confused cone of smoke, the apex of which approaches the orifice of the jet. The cone has not a circular cross-section, but an elongated one, with the major axis in the direction of the sound.

As in the case of gas-flames, the height of the smoke-jets went through maxima and minima as the frequency of the oscillator was increased. The range of sensitivity extended usually from frequencies of about 80 to 900, this being, of course, very much lower than with burning gas-flames. Jets such as W_4, which, when used with gas, showed no sensitivity in the range of the loud-speaker, showed sensitivity when used with smoke in a range from 150–2900. This range included a marked minimum at 2400, a frequency which had been found very effective for gas-flames. On the whole it was found that the frequencies effective with a given jet had to be reduced about ten times if an air-stream was substituted for a burning gas-stream.

An attempt was made to see why the maxima and minima occur. The stroboscope was started, and as its rotation increased the frequency of the loud-speaker was kept in step by turning the vernier of one of the condensers. A nearly static picture of the smoke was thus obtained while the frequency increased and the maxima and minima occurred. At maxima the amplitude of the waves travelling up the stream decreased, and the point of the fork moved consequently to a point further from the orifice, *i.e.*, the stream of smoke increased in height. At minima the amplitude of the waves increased suddenly and the point of the fork moved nearer to the orifice, causing the height of the stream to decrease. But it was impossible to see any reason for the increase or decrease in amplitude, which, in most cases, occurred with the slightest turn of the vernier condenser.

The ratio of the wave-length of the undulations of the stream to its diameter varied, with different jets, from 2·8 to about 7, when the pressure was that for greatest sensitivity.

As can be seen from fig. 14 (Pl. VIII.) the amplitude of the wave in the smoke may double in one wave-length.

The wave-length in the smoke column decreases with decreasing wave-length of the sound, and also with decreasing velocity of the stream, as would be expected. For one jet (W_3) the ratio dropped from 3·7 to 2·9 on reducing the pressure slightly (figs. 16 *a* and 16 *b*, Pl. VIII.).

A series of photographs were taken as follows with one jet, a glass one of the Tyndall type (Pl. IX.) :—

Fig. 17.	Stream at sensitive point but undisturbed.	
,, 18.	Stream disturbed by excess pressure.	
,, 19.	,, ,, noise.	
,, 20.	,, ,, a pure note (frequency 220).	
,, 21.	Case in fig. 18 examined stroboscopically	(frequency 220).
,, 22.	,, fig. 19 ,, ,,	(,,).
,, 23.	,, fig. 20 ,, ,,	(,,).

In each case the photograph was taken from a direction perpendicular to the axis of the jet and to a line joining the loud-speaker with the jet, and the line joining the V-shaped notches in the glass orifice was also perpendicular to this direction.

Fig. 17 shows how the undisturbed stream, on the point of flaring, spreads in a direction perpendicular to the line joining the notches. (The same effect is obtained with jets consisting of a slit.)

Fig. 18 shows that excess pressure caused the stream to spread at a point much nearer the orifice.

Fig. 19 was obtained by tapping on a piece of wood. The stream seems to spread outwards at a much sharper angle.

Fig. 20 shows the effect of a sound of definite frequency. Here the forking is very marked, the smoke being concentrated into the two arms, and in this case more so in one arm than the other. The frequency chosen was the one to which the stream was most sensitive (220).

Fig. 21 is a stroboscopic photograph of the condition shown in fig. 18. The stroboscope was run at the frequency to which the stream responded best (220), but no trace of periodicity can be discerned, nor was it possible to see anything but confused movements at any speed of the stroboscope. Turbulence due to excess velocity does not seem therefore to produce any definite periodicities in the motion of the stream of air.

Fig. 22, a stroboscopic photograph of disturbance produced by tapping on wood (fig. 19), only shows slight signs of periodicity, but actually these are very marked when the

stream is examined visually, although they are changing all the time, and consequently do not show well in a photograph, whose exposure is necessarily as long as half a second.

It appears that the smoke-stream selects from a noise those frequencies to which it is most sensitive.

Fig. 23 shows what is occurring in the case of a disturbance of definite frequency. This has been discussed on p. 182.

It would seem clear from this series of photographs and the observations carried out at the time that they were taken that, although the stream must be on the point of turbulence in order to be sensitive, it is not correct to say, as Tyndall did, that the effect of sound was merely to induce this state of turbulence prematurely. The turbulence due to excess velocity shows no trace of periodicity. On the other hand, the disturbance produced by sound exhibits distinct periodic sinuosities with alternate disengagement of portions of turbulence to left and to right, whether the sound is a pure note or a mixed note, the difference in the two cases being that the sinuosities are much more clearly defined for pure notes.

It is important to show that the behaviour of the stream of air is not due to the presence of particles of smoke. Smoke produced by cigarettes has been found to be composed of very fine particles which readily assume the motion of the air in which they are suspended *. If the smoke-stream be watched carefully as the cigarette is burnt up and for some time afterwards the number of particles decreases gradually to zero, but the sinuosities and the fork remain unchanged until the stream becomes invisible.

Further, by means of exploration with a candle-flame or other small flame in the region above an invisible air-stream it can be shown that the stream has shortened and forked when the sound is maintained.

With gas-streams the gas can be ignited above a sheet of gauze, the column below being unlit. The height of the upper burning portion indicates the sensitivity to sound of the column below. By means of screening portions of the stream from the sound it can easily be seen that it is the unlit column near the orifice which is sensitive. The frequencies to which the unlit stream responds are lower than those to which it responds when ignited, but not so low as in the case of the air-stream. The pressure for sensitivity is also lower. Sensitive flames of the type used by Jordan and McClung †

* E. N. da C. Andrade, Proc. Roy. Soc. A, cxxxiv. p. 445 (1931).

† *Op. cit.*

and by Rayleigh make use of the sensitivity of an unlit column of gas.

It seems evident therefore that the phenomena observed with columns of smoke are not due to, or affected by, the presence of fine particles.

The mechanism of the Rayleigh flame (p. 171) was examined by means of the cigarette smoke. For this purpose the brass tube which screws into the square sound-receiver with mica window was replaced by a pyrex-glass tube which rested firmly in the threaded hole and could be removed rapidly when required.

When the pressure of the smoke-stream was correctly adjusted (which requires as much care as is necessary with coal-gas) it behaved in the same manner as the gas-flame, *i.e.*, a knock on a piece of wood caused the smoke to drop within the top of the tube and, after a few seconds, recover its former position. This was due to the column shortenin and adopting the usual inverted cone appearance. If the noise was a pure note produced by the oscillator the usual wavy motion of the stream occurred, the plane of the undulations being, as usual, that which contained the direction of propagation of the sound. In either case it could be seen that when the stream was disturbed by sound there was a circulation of smoke *downwards* along the walls of the tube. This circulation is troublesome when Rayleigh's flame is used with coal-gas, as it deposits water due to combustion in the upper part of the tube on the jet below. This was observed by Rayleigh *, who got over it by coiling stout copper wire round the jet and connecting it to the hot part of the brass tube at the top.

In every case when the smoke issuing from the top "ducked," due to a note from the oscillator, it was found, by looking through the glass tube or, better, by removing this altogether, that undulations were being produced in the stream causing a certain degree of forking. Further, the pressure was just not sufficient to cause the column to break into turbulent motion, a condition for sensitivity found for all other kinds of jets.

It is quite clear therefore, that the effect of sound on a Rayleigh flame is essentially the same as in the case of ordinary jets. The important difference between Rayleigh's flame and other simple flames is that it enables the sensitiveness of an *unlit* column of gas to be detected. Such unlit columns of gas require a smaller pressure to produce turbulence, and are sensitive to lower frequencies (see p. 181).

* Proc. Roy. Inst. xv. p. 786 (1898).

The low frequency flames of Jordan and McClung * and of Zahradnicek † also make use of columns of gas which are unlit in the lower portion, which is sensitive to sound. Low pressure (and therefore low velocity) streams are very liable to disturbance from draughts, and consequently the sensitive unlit portion has to be protected by a chamber with a window of some substance which will allow the sound waves to enter. The fact that the disturbance of the column of gas is transverse shows that resonance in the tube and chamber surrounding the jet would not aid the sensitivity, but rather hinder it.

Stroboscopic Examination of Gas-jets.

The disturbance in gas-jets produced by pure notes was originally examined by means of a stroboscopic disk before the experiments with smoke-jets. It was, however, impossible to come to any definite conclusion as to what was occurring inside the flame.

With the knowledge gained from the behaviour of smoke-jets, a fresh attempt was made to see whether sinuous waves could not be detected travelling up the column of gas. Such waves were eventually found, and the difficulty of observing them explained. The pressure, and therefore the velocity, of the stream in the case of ignited gas is much higher for the sensitive condition than in the case of unlit streams such as air. Consequently the transverse effect of sound upon the stream does not in general attain such a great amplitude. Further, the outer sheath of the column is at a high temperature, and combustion is taking place within it. This appears to make it insensitive to sound ; at any rate, the wave motion that is observable takes place *inside the outer light blue sheath*, the latter remaining practically steady. The motion of the inner column would be invisible except for the fact that at its outer edge there is a yellow sheath caused by the incomplete combustion of carbon. This, of course, is surrounded on the outside by the light blue sheath, in which more vigorous combustion takes place. It is this yellow portion of the flame which, in the most favourable circumstances, shows first a wavy motion and then forking caused by alternate portions of the undulating column breaking away first on one side and then on the other. This forking does not necessarily break the outer blue sheath, and consequently the flame as a whole does not fork, but merely shortens in height and appears turbulent. In certain

* *Op. cit.* † *Op. cit.*

special cases, however, the motion of the inner column is apparently sufficient to break the outer blue sheath and cause the flame to fork. This was difficult to examine stroboscopically, since forking usually only occurs well at frequencies over one or two thousand. To observe the effects of frequencies as high as this required a very rapid rotation of the stroboscopic disk, and at high speeds a noise was produced which affected the flame considerably and prevented any clear view of the condition of the flame.

After much difficulty a photograph (fig. 28, Pl. X.) was obtained which showed the wave motion and forking of the yellow portion of a flame. In order to do this an unsymmetrical flame was used, in which the yellow portion extended much further down one side, A, than on the other, B (fig. 24, Pl. X.). B is the higher velocity side of the flame, and responds to higher frequencies than the side A. Consequently it was possible to use a frequency which produced disturbance practically entirely in the edge A, and not in the rest of the flame. And further, since the yellow portion at B was higher up than at A, it was possible to examine and photograph the portion A in the direction AA′ (*i.e.*, perpendicular to the line joining speaker and jet) without being troubled by a yellow background at A′, which makes observation very difficult.

A larger aluminium disk was fitted to the stroboscope, and in it were made ten holes of diameter 1 cm. equally spaced and with the ratio of opaque to open portions 7 : 1. The disk was rotated immediately in front of the camera, which had a Zeiss lens of diameter 2·2 cm. In order to get sharpness it was necessary to place a sheet of tin in front of the lens and close to the disk, with a hole in it equal in diameter to those in the disk. The photographs were taken entirely by the light given out by the flame itself, and consequently the exposure necessary was of the order of 15 sec. The stroboscope, however, would not run steadily for such a length of time, and to get over this difficulty there were obviously two methods: one was to let the stroboscope run as nearly as possible at the frequency required, and keep the waves, seen by looking at the flame through the disk, stationary by turning the vernier of one of the condensers in the oscillator, thus altering the frequency very slightly; the other method was to keep the stroboscope constant by "braking" on the rim with the fingers. This latter method was found to be the better, although it required a good deal of practice.

Fig. 25 (Pl. X.) shows the flame in fig. 24 photographed from a direction perpendicular to that of propagation of the sound, and shows the forking taking place at the point A due to sound of frequency 1400 $\sim$.

Fig. 27 (Pl. X.) is a photograph taken at closer range of the same forking. This shows that the outer blue sheath of the flame is displaced slightly in the upper parts of the fork, but is otherwise undisturbed.

Fig. 26 (Pl. X.) shows the appearance of the portion of the flame shown in fig. 27, when there is no disturbance due to sound.

Fig. 28 (Pl. X.) is a photograph of the condition shown in fig. 27 viewed stroboscopically. The yellow portion of the flame enables the undulation, and then forking, of the gas-stream to be seen.

It seems therefore that the same phenomena which occur in air-streams also occur in gas-jets, although in the latter case, partly owing to the higher velocity and partly to the protecting blue sheath in which combustion is taking place, the difficulty of observation is much greater—in fact before the present investigation no record of its observation can be found.

Mathematical Considerations.

The application of hydrodynamical considerations to the problem of the stability of fluid motion in connexion with phenomena of sensitive jets was attempted by Lord Rayleigh in a series of papers *.

Helmholtz had pointed out the possibility of the instability of a surface of separation in a non-viscous liquid where the velocity was discontinuous †, and Rayleigh applied a method due to Kelvin ‡ in order to examine the condition of instability more closely. For the case of a cylindrical jet of fluid, of width $2l$, moving in still fluid of the same density with velocity V, and displaced in such a manner that the sinuosities of its surface are symmetrical about the axis, Rayleigh § shows that

$$h = \mathrm{H}e^{\pm\mu\mathrm{KV}t}\cos\mathrm{K}(\mathrm{V}t - x),$$

where h = amplitude of disturbance perpendicular to the jet at time t. Initially $h = \mathrm{H}\cos\mathrm{K}x$. $\mathrm{K} = 2\pi/\lambda$ and

$$\mu^2 = \mathrm{K}^2a^2\{\log 8/\mathrm{K}a + \pi^{-\frac{1}{2}}\Gamma'(\tfrac{1}{2})\}.$$

* 'Scientific Papers,' arts. 58, 66, 144, 194, 216, 377, and 88.

† Phil. Mag. 4th ser. xxxvi. p. 337 (1868).

‡ *Loc. cit.* xlii. p. 362 (1871).

§ *Loc. cit.* vol. i. art. 58, p. 371

The case of sinuosities whose surfaces are parallel (which is the case with sensitive jets) is worked out for a jet consisting of a plane sheet of fluid. In this case $2l$ is the width of the sheet, and we then have

$$h = \mathrm{H}\epsilon^{\pm(\sqrt{\mathrm{K}l})\mathrm{KV}t} \cos \mathrm{K}(\mathrm{K}l \,.\, \mathrm{V}t - x).$$

The amplitude therefore increases exponentially and disruption will occur the sooner the larger the coefficient KV. As Rayleigh points out in a further paper * this would lead to the conclusion that the instability would increase without limit with K; and since $\mathrm{K} = 2\pi/\lambda$, we should infer that the shorter the wave-length the more sensitive the jet. Experiment shows that this is not the case. The explanation of this contradiction is probably due, he suggests, to the assumption of discontinuity of velocity at the boundary, which in reality is impossible owing to viscosity. There must be a transition layer in which the velocity changes gradually, and when this layer is comparable with the wave-length of the disturbance then the solution ceases to be applicable.

Rayleigh then attempts a two-dimensional treatment of the influence of friction on stratified motion, and comes to the conclusion that, for instance, in the case of a layer of air the thickness would be $1\frac{1}{2}$ cm. at a distance from the orifice numerically equal to the velocity, however thin the layer was initially. He then goes on to attempt a solution for the case of stratification free from discontinuity of velocity, and neglecting the friction. As a result he finds that so far from instability increasing indefinitely with increasing frequency of the disturbed motion, as is the case when the transition is sudden, a diminution of the wave-length below a certain value entails an instability which gradually decreases and is finally exchanged for actual stability. For a jet of given thickness such that the velocity increases uniformly (linearly) from zero at the boundary to a maximum in the centre Rayleigh shows that there should be a maximum instability for a wave-length equal to two and a half times the thickness, and stability should set in again when the wave-length is four times the thickness. The photographs of air-jets, some of which are reproduced in this paper, show that the wave-length may be from at least three to seven times the diameter, and although there is a period of stability when the wave-length is diminished there is another disruptive period when the diminution is carried far

* 'Scientific Papers,' vol. i. art. 66, p. 474.

enough. Obviously the conditions are not so simple as Rayleigh assumes, and this is especially so in the case of gas-jets in which the sensitivity can be shown to belong to one portion of the flame only.

Rayleigh then goes on to show that if the jet is bounded by walls the motion is stable if the velocity curve is convex or concave, *i.e.*, of one curvature throughout. In a further paper * he takes a special case in which the surfaces at which the vorticity ($\frac{1}{2}dU/dy$) changes are symmetrically situated. He is then able to calculate velocity curves between whose limits the motion is unstable. Finally, Rayleigh tackled the problem of motion in two dimensions of an inviscid incompressible fluid *not* enclosed between parallel walls, but he was unable to show any sign of the amplitude of a disturbance tending to increase †. The question has also been discussed by Lord Kelvin ‡.

The essential condition of sensitivity to sound, viz., that the jet should be on the point of flaring, clearly makes any mathematical treatment very difficult, since the flow is on the point of becoming turbulent. This condition does not appear in Rayleigh's calculations, and consequently it is not surprising that his conclusions have little relation to the facts when acoustically-sensitive jets are considered. Further, the exact conditions to which Rayleigh's calculations apply are extremely difficult to gather from his papers ; even Lord Kelvin found some of them "very difficult reading, in every page, and in some ∞*ly* difficult §."

The Criterion for Turbulence.

Although mathematical calculations throw very little light on the nature of the instability underlying the behaviour of sensitive flames, a few general observations may be made which involve the criterion for turbulence.

First of all, there is no doubt at all that an essential condition for sensitivity is that the flame shall be on the point of "flaring," *i. e.*, that the stream is about to break up owing to turbulence of some kind. The vibrations of sound are incapable of producing a transverse oscillation of the column of gas, unless it is near the turbulence point. Consequently the initial stages of turbulence must favour the production of undulations in the column. Why this

* *Loc. cit.* iii. art. 144, p. 17.
† *Loc. cit.* iv. art. 216, p. 203.
‡ Phil. Mag. xxiv. pp. 188, 272 (1887).
§ Quoted by Rayleigh, *loc. cit.* p. 269.

should be so is not at all clear, and will be the subject of further research. If it is granted, however, that turbulence is in some way an essential to sensitivity, then by the use of Osborne Reynolds' criterion we may explain a number of facts that have been described in the preceding pages. Assuming for simplicity that the jet of gas behaves like a stream in a circular pipe, we should have for the criterion of turbulence that

$$\text{velocity} \not> \frac{K}{r} \cdot \frac{\eta}{\rho},$$

where K is a constant, r is the radius of the tube, η is the viscosity, and ρ is the density. For a given jet the velocity for turbulence depends only on the kinematic viscosity η/ρ. The fact that the boundary is not rigid and that the surrounding air is dragged upwards with the stream to a certain extent will make the effective value of r in the formula greater than the radius of the jet at the orifice.

Considering the radius term r first of all, it is clear that streams of small diameter can reach a greater velocity before the turbulent state is reached than streams of larger diameter. In other words, the pressure for sensitivity (*i. e.*, turbulence) will be higher the smaller the diameter of the jet. This was found to be the case. Further, it was found that for a given gas the higher velocity jets were sensitive to higher frequencies. Let us now assume that the velocity necessary for turbulence is connected in much the same way with the frequency sensitiveness when we pass from one gas to another. If now the radius is kept constant the velocity for turbulence will be greater the greater the kinematic viscosity, and, if our assumption is correct, the frequency also. We should expect then that unlit coal-gas would be sensitive to slightly higher frequencies than air, since although the ratio of the densities is 1 : 2 the ratios of the viscosities is nearer 2 : 3. This also was found to be the case (p. 185). Again, if we neglect complications arising from combustion at the outer surface only, the burning gas will have much greater kinematic viscosity, since the viscosity increases and the density decreases with temperature. If the average temperature of the flame is taken as 1500° C.*, the ratio of the kinematic viscosity to that at 15° C. is approximately 28. The velocity for turbulence will clearly be much higher, and consequently the frequency sensitivity also. This again is what is observed. A quantitative

* A. T. Jones, *loc. cit.*

confirmation can be sought by considering water-jets in water. Here the kinematic viscosity is reduced about ten times compared with air, and if we invoke the laws of dynamical similarity, as was done by Rayleigh *, the frequency sensitivity should be reduced in the same ratio. This is well borne out by the facts :—

Most sensitive frequencies for air......	220–700	
,, ,, ,, ,, water ..	20– 50	(Rayleigh).

It is clear, however, that simple considerations such as these only have a very subsidiary bearing on the phenomena of sensitive jets. A close examination of the photographs shown in figs. 14 and 15 (Pl. VIII.) shows that the motion of the smoke column, even before it bifurcates, is very complex. The small wisps of smoke left behind in the concave parts of the undulation, which later assume a hook-like appearance, make it very probable that the sinuosity of the column is in reality due to incipient vortices travelling alternately up the opposite sides of the column. As the vortices grow, they give a more and more hook-like appearance to the boundary of the column, until finally they separate and pass alternately up the arms of the forking stream.

There is a very close resemblance between the appearance of a column of smoke which is forking owing to the action of sound and a column of smoke from a slit which is producing edge-tones. This can be seen by comparing the diagrams given by Carrière in a paper on edge-tones † with figs. 14 and 15 (Pl. VIII.). The same hook-like appearance of the boundary occurs in the undulating column before the vortices are fully formed. At the edge the vortices separate, thus producing a fork, along the arms of which they pass alternately in a manner similar to that observed in a sensitive jet which is forking under the influence of sound. The exact mechanism of the production of edge-tones is, however, still very obscure ‡.

Summary and Conclusion.

The sensitivity to sound of gaseous streams issuing into air through jets of various kinds has been examined. If the size of the orifice be below about $\frac{1}{2}$ mm. in diameter the stream is short and the amount of "ducking" very small.

* *Loc. cit.* ii. p. 273.

† *J. de Phys.* vi. p. 52 (1925).

‡ *Cf.* E. G. Richardson, "Edge-Tones," Proc. Phys. Soc. xliii. p. 397 (1931).

If the diameter of the jet exceeds about $\frac{1}{2}$ cm. the stream reaches to a great height, is very susceptible to draughts, and suffers from periodic bulbous disturbances symmetrical about the vertical axis of the jet, which travel upwards and help to mask the effect produced by sound. In between these limits all streams show some sensitivity to sound. This is always most marked when the stream is on the point of "flaring." If any roughness or obstruction in the jet causes premature "flaring," *i. e.*, turbulence at a lower velocity, then the sensitivity of the stream is markedly impaired and the range of frequencies to which it responds is lowered. If an obstruction near the orifice has the effect of splitting the column into streams of smaller diameter and higher velocity, then in this case the range of frequencies to which the flame is sensitive is displaced in the direction of greater frequency.

The position and construction of the tube supplying gas to the jet affects the sensitivity in so far as it can alter the way in which the gas streams out of the orifice. In practice the best arrangement has to be found by trial, and consequently rubber tubing serves this purpose best. If the jet contains a plug of cotton wool, the position and nature of the supply tube has no effect on the behaviour of the stream issuing from it. Taps and clips can be used provided that the gas in passing them does not produce sounds to which the stream is sensitive.

As regards the frequencies which were most effective, it was invariably found that for a given gas the frequencies producing maximum disturbance and the frequencies producing minimum disturbance were definite and did not vary with the kind of jet used: they are constants for any particular gas. The values for a hydrogen flame could not be distinguished from those for an ordinary coal-gas flame.

The *range* of frequency to which gaseous streams are sensitive varies with the gas used and with the size and nature of the jet employed. With air the range is quite low (80–900), with unlit gas somewhat higher, and with ignited coal-gas or hydrogen the range extends (with different jets) from 500 to over 18,000. The size and nature of jet affect the range by altering the velocity which leads to turbulence. If a large number of circular orifices are examined it is quite clear that the jets with high-frequency ranges are the ones where the velocity for sensitiveness is high and the ones with low-frequency sensitivity are those with low velocity. Apparent exceptions to this rule are

caused by minute irregularities in the orifice and to the position of the rubber supply tube.

The actual effect of the sound waves on a gaseous column is to cause regular undulations to proceed from the jet up the column, the upper part of the column breaking into turbulence. The wave-length of these undulations depends, as would be expected, on the velocity of the column and the frequency of the sound. The most sensitive jets are those which produce a stream which tends to break up naturally into turbulent motion in one plane rather than into turbulence symmetrical about the axis. Such jets show marked directional properties, and when the turbulent motion takes place in a plane which includes the direction of propagation of the sound wave the undulations in the lower part of the column increase to such amplitude that the stream forks. In a direction perpendicular to this the jets exhibit practically no sensitivity at all.

Although the condition of being on the point of breaking down into turbulent motion is a criterion of sensitiveness to sound for all jets, and must therefore favour the production of sinuosities in the stream, nevertheless the turbulence caused by excessive velocity does not exhibit any sinuosity or periodicity of any kind. The sound does not, as has been generally assumed, precipitate the state of turbulence into which the stream is about to fall—it produces a state of periodic disturbance which is quite distinct.

It is hoped that further research will enable the effect of sound on the motion of a gaseous column to be observed in greater detail, thereby not only throwing more light on the nature of the instability underlying the phenomenon of sensitive flames, but also perhaps on what appears to be an allied phenomenon, that of the production of edge-tones.

The author wishes, in conclusion, to express his great indebtedness to Professor E. N. da C. Andrade, not only for initiating the research on sensitive jets, but also for his continued help and interest in it. The author would also like to thank Dr. E. G. Richardson and Dr. R. E. Gibbs for their part in many valuable discussions which have taken place during the course of the work.

Carey Foster Laboratories,
University College, London.
July 1931.

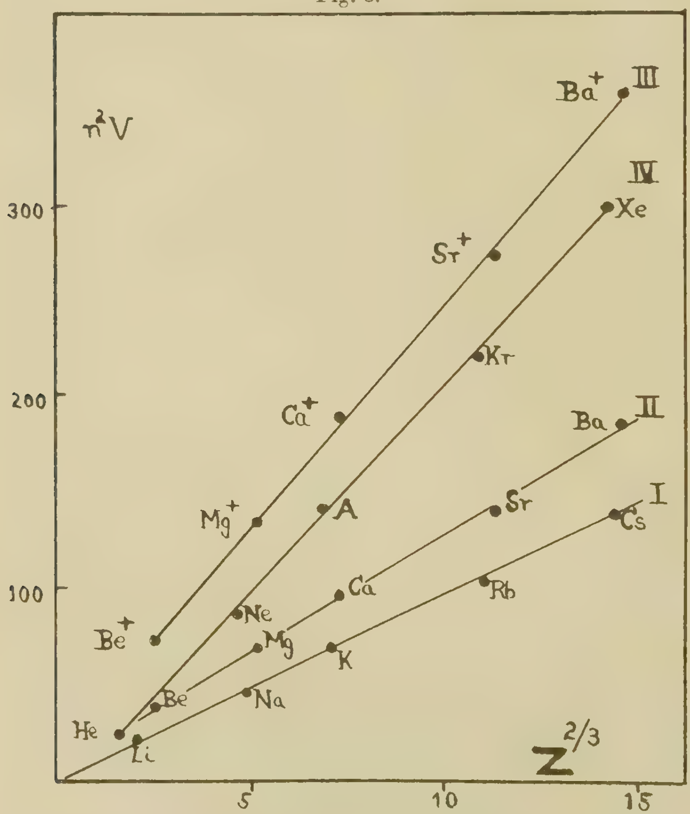
XIV. *The Electronic Energy Levels of the Elements with special reference to their Connexion with the Sizes and Electronic States of Atoms in Metallic Crystals. A Correction.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

IN my paper, which was published under the above title in volume xi. of your Journal, some errors occur in fig. 8 on page 671. The numbers given in Table IV. below

Fig. 8.



In this figure n^2V is plotted against $Z^{2/3}$, where V is the ionization potential necessary to remove an electron of quantum number n. The curves marked I and II refer to the first ionization potentials of the alkali and alkaline earth sub-groups, whilst curve III refers to the second ionization potential of the alkaline earth group, and curve IV to the first ionization potential of the rare gas group. The data are given in Table IV.

this figure are correct, except for a misprint in the case of Krypton for which $Z^{2/3}$ equals 10·9 and not 13·9 as printed. In the drawing of the diagram some mistakes appear to have been made, and I can only suggest that the paper must have slipped when the diagrams were transferred from squared paper to Bristol board.

These mistakes are corrected in the diagram printed herewith, and it will be seen that they in no way vitiate the general conclusion of an approximately linear relation between n^2V and $Z^{2/3}$. When the corrections are made it will be found that the "fit" of the points to the best straight lines is slightly worse for line III. but equally good or even better in lines I., II., and IV. The original figure can easily be corrected by means of the data given in Table IV. on page 671, and I can only apologize for any confusion which may have been caused by my not noticing such obvious errors when correcting the proofs.

Yours faithfully,
W. Hume-Rothery.

XV. *Notices respecting New Books.*

Past Years: an Autobiography. By Sir Oliver Lodge. [Pp. 364, with portraits.] (London: Hodder and Stoughton, 1931. Price 20*s*. net.)

THIS is a volume of reminiscences rather than an autobiography, because it does not preserve strict historical order, but is divided into sections between which there is considerable overlapping. This makes it difficult sometimes to picture the precise order of development. The final impression produced is that of a man with many interests. The main one no doubt has been Natural Philosophy, obtained first by devouring Brewer's 'Child's Guide to Knowledge,' and by watching steam-engines, and from the tuition, by an aunt, in Astronomy. He learned nothing of science at school, and on leaving he assisted his father in business in the Potteries. A visit to an aunt in London forms a turning-point in his life; she encouraged him to attend lectures, including some by Tennant at King's College on Geology, who gave him a ticket for a course on Heat by Tyndall illustrated by a great number of experiments. These were an "eye-opener," and he absorbed the course eagerly. He attended others on various subjects at University and King's Colleges, including one by Daniell, and he at once began making experiments at home and devouring Smiles' 'Self Help' and encyclopædias; and going in for Science and Art examinations, studying Chemistry with

Frankland (or his assistant, Valentine), and, a little later, coming under the influence of Carey Foster, Clifford, and Henrici; teaching Mathematics, Physics, and Chemistry, and taking degrees.

Prior to (and leading up to) some of these events (while travelling for his father) he had attended the Bradford meeting of the British Association (1873). In Section A there were H. J. S. Smith and Glaisher; Clerk Maxwell lectured on Molecules and W. C. Williamson on Coal Plants. The whole meeting was an experience never to be forgotten, and he never failed to attend the annual meeting until the Association went to South Africa in 1905. The Bradford meeting was the means of him breaking away from business. Soon afterwards his own publishing of original work began.

His first (and only) Professorship began in 1881 at Liverpool in the newly founded University College, and lasted till 1900, when he was appointed Principal at Birmingham. His laboratory in Liverpool was in an old building. "There was nothing sacred about the walls." They could be plugged and pulled about and holes cut in them wherever wanted. It was here that Lodge's main experimental work was done.

I first saw Lodge at a soiree, in connexion with the newly founded University College at Liverpool, somewhere about 1885. He there showed his fog-clearing experiment—in a bell-jar (p. 180). It was never a success on a large scale, but he and two of his sons have turned it to good use in connexion with smoke-dust deposition in certain factory processes (pp. 180, 248). Of his research work on oscillatory electric discharges I have a closer acquaintance. He scarcely emphasizes enough the meagre provisions at Liverpool for research. This work was largely done in the Physics Theatre, which was used for Junior as well as Senior Lectures. The lecture table had to be cleared as much as possible for these important events, research apparatus being pushed to one side. The room was surrounded by wires (stretched on posts) on which electric waves could be excited. This was all to the good so far as higher lectures were concerned, because an experimental discovery could be made on one day and shown to honours students the following day without much trouble. Probably no senior students have ever been more fortunate in that respect. They saw most of his earlier experiments (described in Chap. XVII.) "hot from the furnace" in 1888–1890.

He says little about his routine teaching. He probably had too much of it to be able to regard it with enthusiasm when there were so many more important things to do; yet he is a born expositor, whether it be in a Junior Lecture or a popular lecture, or in summing up discussions such as that on the Universe at the recent meeting of the British Association. He knows how to supply the element of surprise. I have heard him at the Royal Institution emphasizing the need of an Æther by means of an experiment. "I wave this rod over the weight on the table; you

do not expect to see it move." "But it does," he added, as the weight rose, lifted by an invisible connecting thread.

It is impossible in this place to detail all the phases of his active life. One, however, must be briefly mentioned. A student in his Matriculation Mechanics class—Edmund Gurney—introduced him to F. W. H. Myers.

Through Myers and Gurney, and also Barrett of Dublin, he became interested in investigations on psychical matters. The result is recorded in Chapters XXII.–XXIV. Physicists may regret that he was thus distracted from his physical investigations. On the other hand, others may rejoice that one so thoroughly acquainted with physical methods should approach psychical investigations in a sympathetic spirit. These enquiries lie outside the scope of this Journal.

Lodge has always been in favour of the use of models in explaining physical phenomena. "The abstract method of treatment at present in vogue probably represents a phase through which science has to go. It is being conducted through the haze with great ability; but in time, I believe, it will emerge on the other side and become intelligible once more, with an added perception of reality and a clearer conception of its working."

We have said nothing about the personal relations with his family and with others. The account of these fills a considerable part of the volume. The picture drawn is a very attractive one.

Dielectric Constant and Molecular Structure. By C. P. Smyth. (The Chemical Catalog Company, New York, 1930. 4 dollars.)

This book is an American Chemical Society Monograph, issued by the Chemical Catalog Company of New York. By arrangement with the Inter-allied Conference of Pure and Applied Chemistry which met in London and Brussels in July 1919, the American Chemical Society undertook the production and publication of Scientific and Technologic Monographs on chemical subjects, and a long series of important publications have already appeared.

In this volume the author succeeds in correlating and organizing the relevant body of facts and the theories in terms of which they are interpreted. The book is noteworthy for the masterly way in which these theories are explained and critically examined. The summary of literature and the account of the methods of investi gation cannot but be of service to those specially interested in this rapidly growing subject.

Mathematics. By B. B. Low. (Longmans, Green and Co., 1931. Price 12*s*. 6*d*.)

This is a text-book designed for the use of those studying engineering or chemistry. It covers the elements of trigonometry, coordinate geometry, differential and integral calculus, and differential equations. The treatment throughout is elementary and adequate for its purpose. Mathematical tables are included.

Operational Methods in Mathematical Physics. By HAROLD JEFFREYS, F.R.S. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 23.) Second Edition. (Cambridge University Press, 1931. Price 6*s.* 6*d.*)

IT is gratifying to see that this excellent Tract, first published in 1927, has already run into a second edition. There are few changes: the chapter on Bessel Functions has been re-written, and a discussion of the transmission of a simple type of telegraphic signal along a cable has been added. It remains only to repeat that this Tract can be thoroughly recommended to those who have to do with the differential equations of physics.

James Clerk Maxwell: a Commemoration Volume, 1831–1931. [Pp. 146; two portraits.] (Cambridge University Press, 1931. Price 6*s.* net.)

THIS commemoration volume is a sheer delight. It records why representatives of many nations assembled at Cambridge to celebrate the centenary of Maxwell's birth. It is introduced by a biographical essay by Sir J. J. Thomson, and followed by essays by Max Planck, Albert Einstein, Sir J. Larmor, Sir James Jeans, William Garnett, Sir Ambrose Fleming, Sir Oliver Lodge, Sir Richard Glazebrook, and Sir Horace Lamb. Garnett was the first Demonstrator in the newly opened Cavendish Laboratory; Fleming and Glazebrook were amongst the early workers there, and Lamb attended some of Maxwell's lectures. The book is consequently punctuated with personal touches. Max Planck deals with his influence on theoretical physics in Germany and Einstein on his influence on the development of the conception of physical reality. Garnett, Fleming, and Glazebrook show most interest in the laboratory work. Lodge describes how Maxwell's speculations on electromagnetic waves led up through Hertz and his own experiments to their experimental production, and ultimately through Marconi and others to wireless telegraphy.

The book should be in the hands of every physicist and physical student.

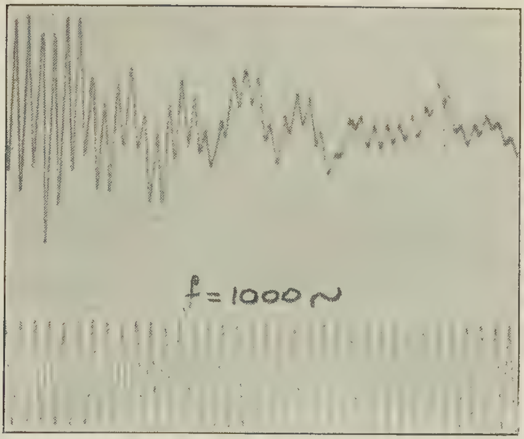
The Mysterious Universe. By Sir JAMES JEANS, F.R.S. [Pp. 142.] (Cambridge University Press. Price 2s.)

THIS is a revised edition of a book the sale of which has run into six figures. It is not a mere reprint. The first four chapters have been brought up to date and any ambiguities detected have been removed. It is a book which everyone should read whether he agrees with the point of view taken or not. It will show him the direction in which speculative scientific thought is tending at the present time.

The book has not only been improved but its price is also less.

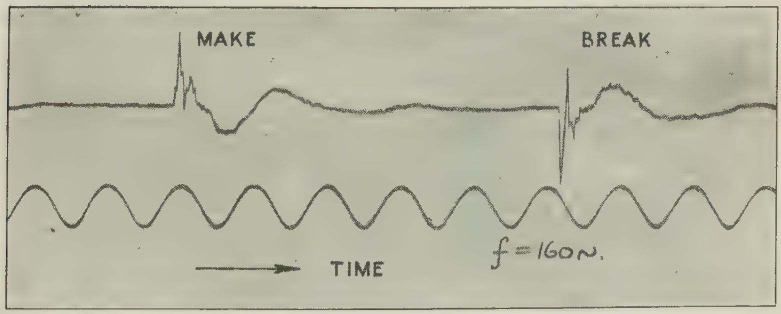
The Editors do not hold themselves responsible for the views expressed by their correspondents.

Fig. 7.



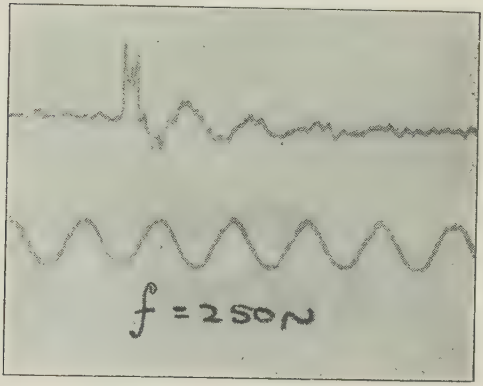
Impulse record of acoustic output from reed-driven paper disk. There are two main oscillations (1) 260 cycles, (2) 1200 cycles which correspond to the frequencies found from the sand-patterns of fig. 6.

Fig. 19.



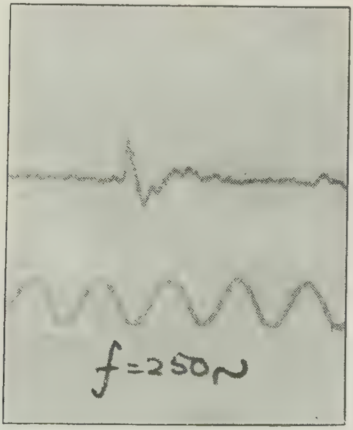
Impulse record of permanent magnet commercial reproducer with leather surround and diaphragm 7·5 cm. radius. The diaphragm oscillates on the surround at 80 cycles per second, since the magnetic field is inadequate to overcome the strong control thereof.

Fig. 15 *a*.



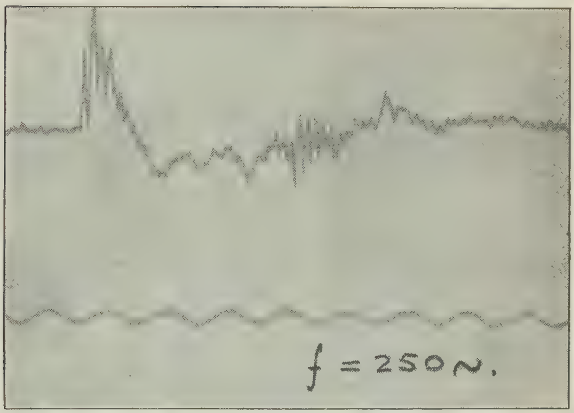
Impulse record of W. E. Co. 555 W. moving-coil receiver without horn.

Fig. 15 *b*.



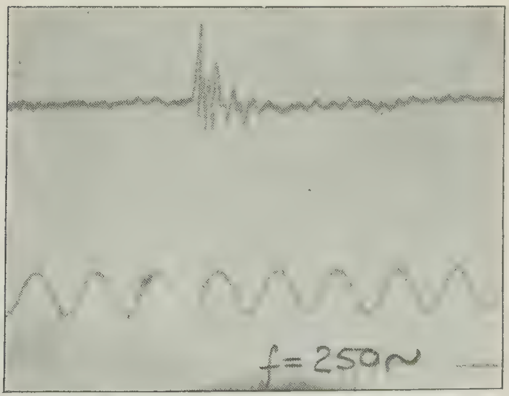
As at fig. 15 *a*, with straight exponential horn 5 feet long.
The damping effect of the horn is evident.

Fig. 16*a*.



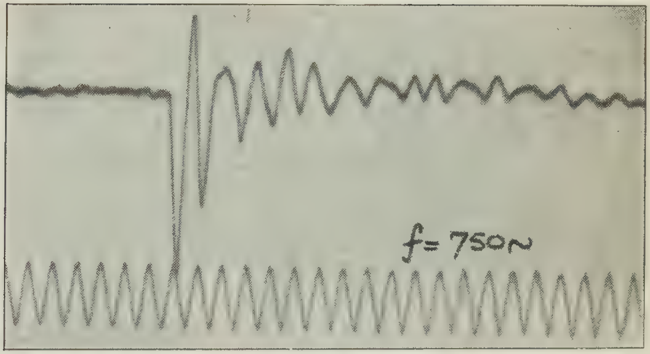
Impulse record of standard coil-driven diaphragm with tight rubber surround. The 33 ~ oscillation is the diaphragm moving as a whole on the surround. The 200 ~ oscillation is due to the surround *per se*, whilst the 2000 ~ oscillation is the second symmetrical mode of the diaphragm. The coil (40 turns) was transformer coupled, but the e.m. damping was reduced owing to the field being 0·6 its normal value.

Fig. 16*b*.



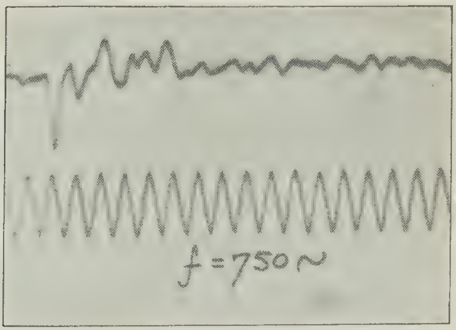
Impulse record as at fig. 16 *a*, after removal of rubber surround. Only the 2000 ~ oscillation due to the main symmetrical mode remains.

Fig. 17 *a*.



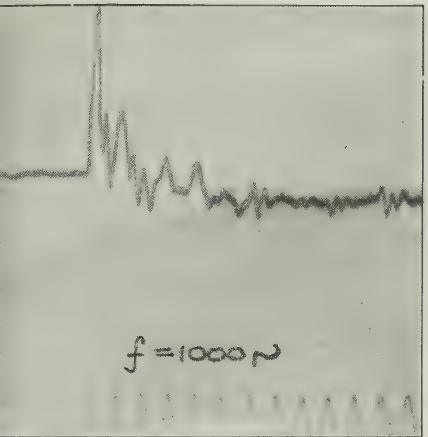
Impulse record of free-edge coil-driven paper diaphragm 16·7 cm. radius, apical angle 160°, taken with microphone on axis at a distance of 23 cm. from mouth of diaphragm. The natural frequency is that of the second symmetrical mode.

Fig. 17 *b*.



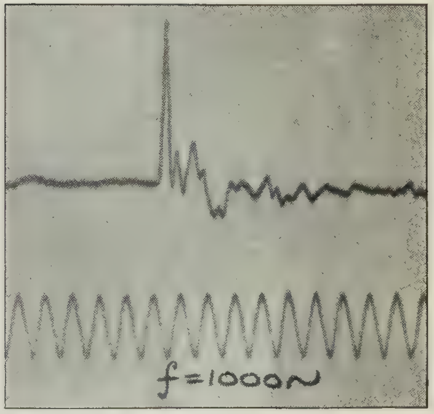
As at fig. 17 *a*, but microphone 25 cm. away from axis.

Fig. 18 *a*.



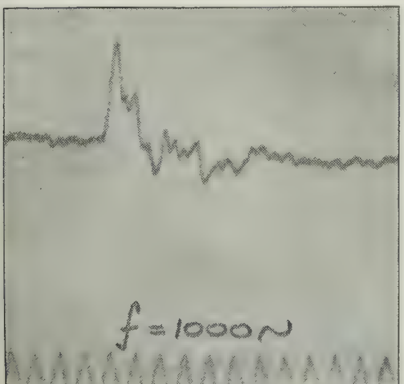
pulse record of diaphragm of fig. 4 taken with microphone on axis at a distance of 23 cm. from mouth of cone. The motion of the diaphragm on the surround ($f = 18{\cdot}7 \sim$) and of the surround *per se* ($129{\cdot}5 \sim$) is aperiodic. The symmetrical mode recorded is about 2600 cycles per second.

Fig. 18 *b*.



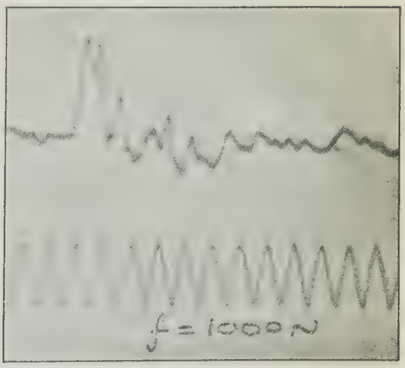
As at fig. 18 *a*, but microphone 70 cm. from mouth of cone.

Fig. 18 *c*.



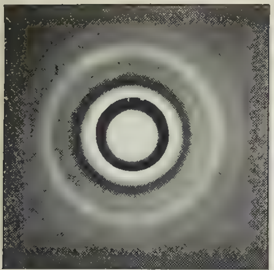
As at fig. 18 *a*, but microphone 25 cm. away from axis.

Fig. 18 *d*.



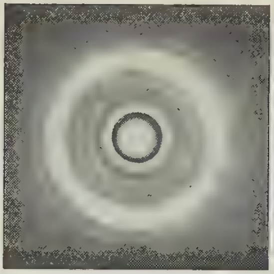
As at fig. 18 *a*, but microphone 28 cm. in front and 40 cm. away from axis.

FIG. 1.



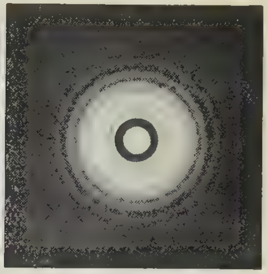
1/13·5 *h-p.*

FIG. 2.



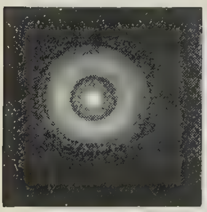
1/6·2 *h-p.*

FIG. 3.



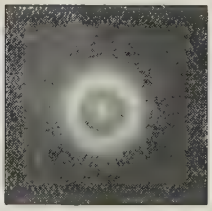
1/3·5 *h-p.*

FIG. 4.



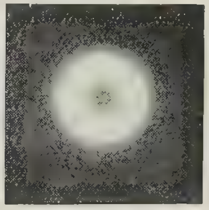
$\frac{1}{2}$ *h-p.*

FIG. 5.



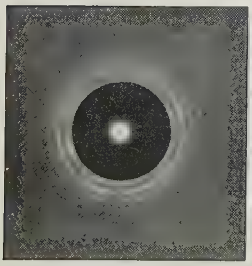
1 *h-p.*

FIG. 6.



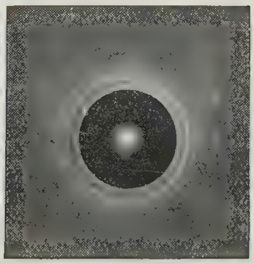
1$\frac{1}{2}$ *h-p.*

FIG. 7.



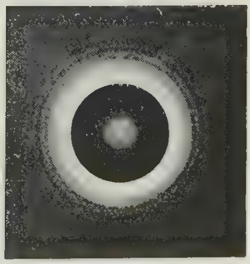
$\frac{1}{2}$ *h-p.*

FIG. 8.



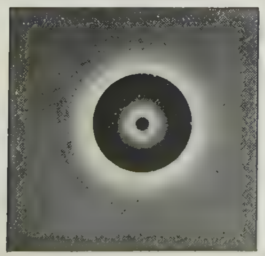
1 *h-p.*

FIG. 9.



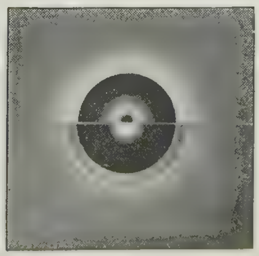
1$\frac{1}{2}$ *h-p.*

FIG. 10.



2 *h-p.*

FIG. 11.



2 and $\frac{1}{2}$ *h-p.*

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

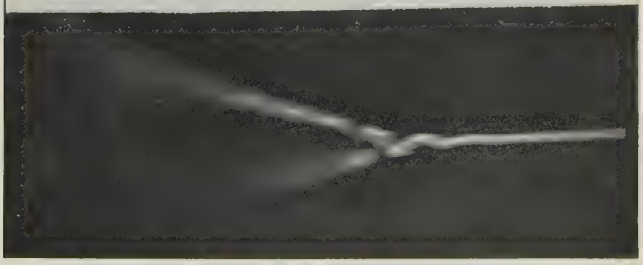


Fig. 14.

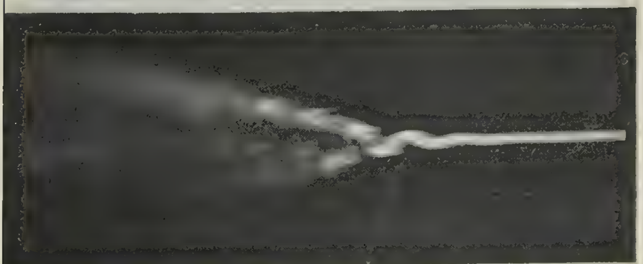


Fig. 15.

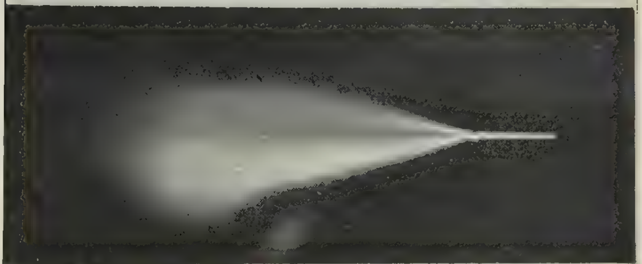


Fig. 16 *a*.

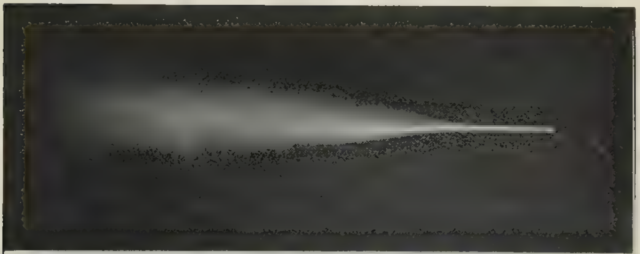


Fig. 16 *b*.

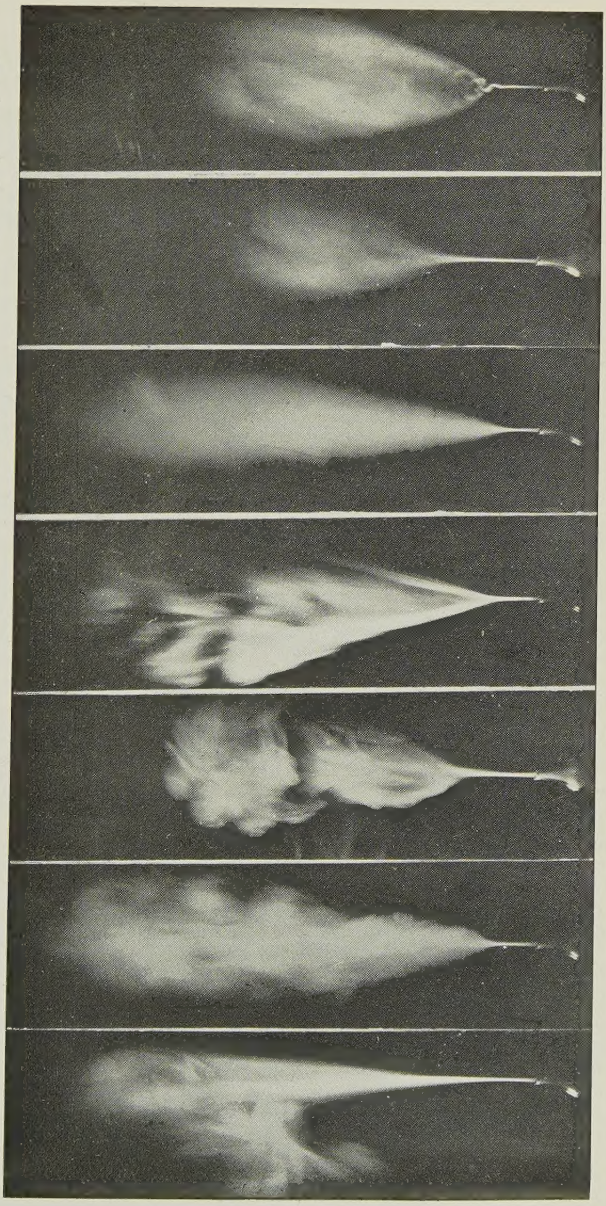


Fig. 23.

Fig. 22.

Fig. 21.

Fig. 20.

Fig. 19.

Fig. 18.

Fig. 17.

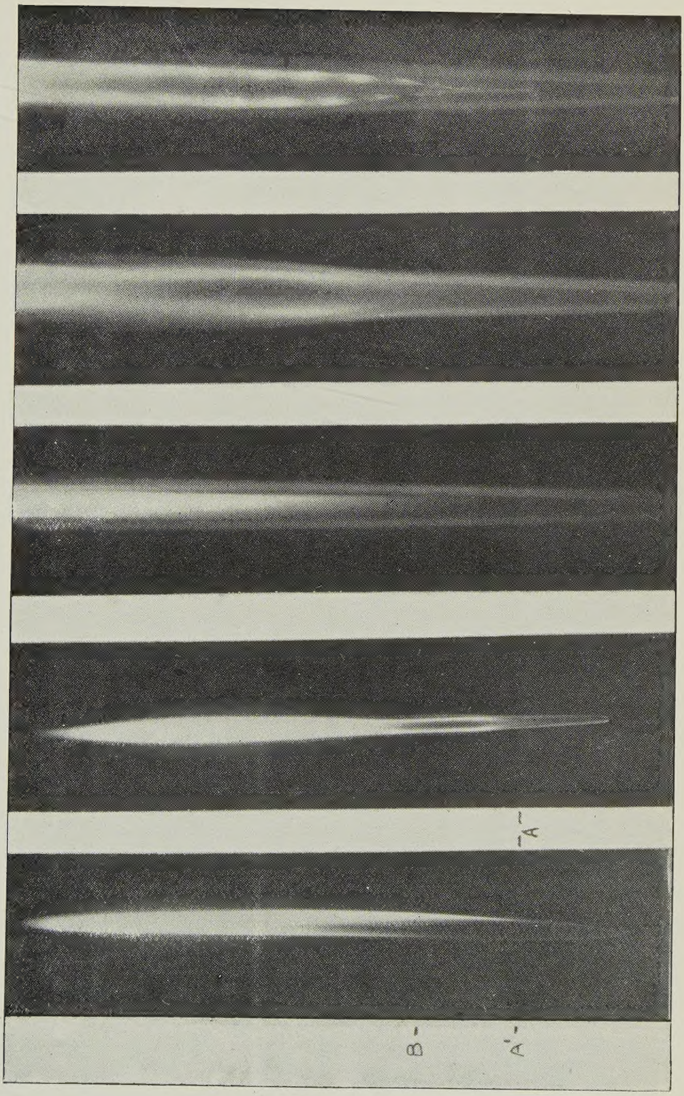


Fig. 28.

Fig. 27.

Fig. 26.

Fig. 25.

Fig. 24.